MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Lieut.-General J. F. TENNANT, C.I.E., R.E., F.R.S., President, in the Chair.

Thomas Steele Sheldon, M.B.Lond., Parkside, Macclesfield, was balloted for and duly elected a Fellow of the Society.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended :—

Rev. John Mitchell, B.D., 57 Parkgate Road, Chester (proposed by John Hartnup).

The following were proposed by the Council as Associates of the Society :—

Lewis Boss, Dudley Observatory, Albany, N.Y., U.S.A.;
A. Cornu, Paris ;
C. Souillart, Lille, France.

Comparison of the Right Ascensions of Clock Stars in the Greenwich Ten-Year Catalogue for 1880 *with the Fundamental Catalogues of the American Ephemeris and of the Astronomische Gesellschaft.* By Professor Simon Newcomb.

The fundamental catalogues with which this comparison is made are found in the following three publications :—

(*a*) *Catalogue of* 1098 *Standard Clock and Zodiacal Stars; Astronomical Papers of the American Ephemeris*, vol. i.

(*b*) *Fundamental-Catalog für die Zonen-Beobachtungen am nordlichen Himmel. Herausgegeben im Auftrage der Zonen-Com-*

mission der Astronomischen Gesellschaft, von A. Auwers. Pub. XIV. *der Astronomischen Gesellschaft.*

(*c*) *Vorläufiger Fundamental-Catalog für die südlichen Zonen der Astronomischen Gesellschaft.* Von A. Auwers. *Ast. Nachr.* No. 2890, vol. cxxi. p. 145.

I have ignored the preliminary fundamental catalogue given in Publication XVII., *Astronomische Gesellschaft,* supposing it to be superseded by the last catalogue (*c*) just quoted. And I have devoted a separate column to the latter because it gives revised positions of a few stars of (*b*).

The latest Greenwich observations employed in these fundamental catalogues were those for the year 1876. Such at least is the case for the first two, and I infer the same to be true of the last, because later Greenwich observations are not mentioned in the list of authorities on which it rests. The observations on which the Ten-Year Catalogue was formed began in 1877, so that its results are quite independent of the catalogues compared. Some interest may therefore attach to a comparison of the authorities, as affording an index to the degree of precision attained in the right ascension of fundamental stars, and suggesting special points to be attended to in still further improving these right ascensions. In the following exhibit the second column contains the right ascensions, given on pp. 56 and 57 of the Ten-Year Catalogue, as based on 12-hour groups. The three following columns (*a*, *b*, *c*) contain the decimals of the seconds of right ascension from the three fundamental catalogues, taken in order. In each case the positions for 1880·0 have been derived from those of the catalogue by using the values of the proper motions given therein.

The three following columns contain the apparent corrections in right ascension given by the Greenwich results :—

Name of Star.	Right Ascensions for 1880·0. Ten-Year Cat.	(*a*)	(*b*)	(*c*)	App. Corrections. (*a*)	(*b*)	(*c*)
	h m s	s					
α Androm.	0 2 11·186	·210	202	...	−24	−16	...
γ Pegasi	0 7 3·408	·455	·449	...	−47	−41	...
ι Ceti	0 13 18·801	...	·812	·804	...	−11	− 3
44 Piscium	0 19 15·070	·079	...	...	− 9	...	...
12 Ceti	0 23 54·846	·878	...	·894	−32	...	−48
ε Androm.	0 32 12·961	...	·993	...	...	−32	...
β Ceti	0 37 33·904	·947	...	·918	−43	...	−14
δ Piscium	0 42 27·397	·419	...	...	−22	...	...
20 Ceti	0 46 52·483	·483	...	...	0	...	...
μ Androm.	0 50 5·711	...	·826	...	...	−115	...
ε Piscium	0 56 42·932	·952	·956	...	−20	−24	...
β Androm.	1 3 0·932	·996	·001	...	−64	−69	...
ζ^1 Piscium	1 7 27·711	·717	...	...	− 6	...	...

Name of Star.	Right Ascensions for 1880·0. Ten-Year Cat.			(a)	(b)	(c)	App. Corrections. (a)	(b)	(c)
	h	m	s	s					
θ Ceti	1	18	1·479	·519	·544	·532	−40	−65	−53
η Piscium	1	25	3·758	·805	·796	...	−47	−38	...
ν Piscium	1	35	11·195	·234	...	·216	−39	...	−21
ο Piscium	1	39	3·432	·477	·456	...	−45	−24	...
β Arietis	1	48	0·727	·762	·760	...	−35	−33	...
α Arietis	2	0	24·611	·646	·640	...	−35	−29	...
ξ^1 Ceti	2	6	38·408	·452	...	...	−44	...	...
67 Ceti	2	10	59·876	...	...	·901	...	...	−25
ξ^2 Ceti	2	21	46·754	·806	·787	...	−52	−33	...
ν Ceti	2	29	34·633	...	...	...	...	...	...
δ Ceti	2	33	19·903	...	·951	·926	...	−48	−23
γ^2 Ceti	2	37	4·961	·998	·002	...	−37	−41	...
σ Arietis	2	44	52·081	·122	...	...	−41	...	...
ε Arietis	2	52	21·081	·126	...	...	−45	...	...
α Ceti	2	56	0·412	·440	·420	·424	−28	− 8	−12
δ Arietis	3	4	46·086	·113	...	...	−27	...	...
τ^1 Arietis	3	14	18·005	·055	...	...	−50	...	...
ο Tauri	3	18	21·349	...	·394	...	...	−45	...
f Tauri	3	24	14·890	·921	·930	...	−31	−40	...
ε Eridani	3	27	16·603	·626	·622	·642	−23	−19	−39
11 Tauri	3	33	36·336	·361	...	...	−25	...	...
δ Eridani	3	37	29·996	...	...	·020	...	...	−24
η Tauri	3	40	21 148	·149	·160	...	− 1	−12	...
γ^1 Eridani	3	52	25·847	·887	...	·848	−40	...	−1
A^1 Tauri	3	57	36·109	·144	...	...	−35	...	...
ω^1 Tauri	4	2	10·550	·586	...	...	−36	...	...
o^1 Eridani	4	6	0·437	...	...	·481	...	...	−44
γ Tauri	4	12	57·876	·924	·913	...	−48	−37	...
ε Tauri	4	21	36·570	·619	·626	...	−49	−56	...
Aldebaran	4	29	2·111	·144	·124	...	−33	−13	...
τ Tauri	4	35	2·592	·609	...	...	−17	...	...
μ Eridani	4	39	30·125	...	·138	·146	...	−13	−21
ι Aurigæ	4	49	10 753	·815	·822	...	−62	−69	...
ε Leporis	5	0	22·861	...	...	·890	...	...	−29
Rigel*	5	8	46·253	·267	·256	·218	−14	− 3	(+35)
β Tauri	5	18	42·382	·407	·398	...	−25	−16	...
δ Orionis	5	25	52·519	·577	·554	...	−58	−35	...

* Probably there is a typographic error in (*c*).

Name of Star.	Right Ascensions for 1880·0. Ten-Year Cat.	(a)	(b)	(c)	App. Corrections. (a)	(b)	(c)
	h m s	s					
α Leporis	5 27 26·260	·272	...	·267	−12	...	− 7
ε Orionis	5 30 7·444	·471	·449	·466	−27	− 5	−22
α Columbæ	5 35 18·225	·286	...	...	−61	...	...
κ Orionis	5 42 3·905	...	·898	·900	...	+ 7	+ 5
α Orionis	5 48 40·505	·516	·504	...	−11	+ 1	...
1 Geminor.	5 56 49·502	·550	...	...	−48	...	...
ν Orionis	6 0 43·184	·271	...	...	−87	...	...
η Geminor.	6 7 38·018	·072	·052	...	−54	−34	...
μ Geminor.	6 15 41·995	·059	·042	...	−64	−47	...
β Canis Maj.	6 17 24·883	...	...	·915	...	...	−32
ν Geminor.	6 21 50·238	·259	...	...	−21	...	...
γ Geminor.	6 30 46·733	·768	·771	...	−35	−38	...
ξ Geminor.	6 38 33·222	...	·242	...	...	−20	...
θ Can. Maj.	6 48 36·824	...	...	·892	...	...	−68
ε Can. Maj.	6 53 54·563	·612	...	...	−49	...	...
ζ Geminor.	6 56 59·424	·496	·476	...	−72	−52	...
γ Can. Maj.	6 58 19·738	...	...	·771	...	...	−33
51 Geminor.	7 6 28·770	·774	...	...	− 4	...	...
δ Geminor.	7 12 57·306	·337	·320	...	−31	−14	...
β Can. Min.	7 20 38·533	...	·564	...	...	−31	...
Castor	7 26 56·497	·542	...	...	−45	...	...
Procyon	7 33 1·109	·185	·139	·107	−76	−30	+ 2
Pollux	7 37 58·265	·298	·289	...	−33	−24	...
ξ Argûs	7 44 14·861	...	...	...	...	...	...
6 Cancri	7 56 8·739	·808	...	...	−69	...	...
15 Argûs	8 2 25·981	·026	...	·025	−45	...	−44
β Cancri	8 10 0·367	...	·402	...	...	−35	...
d^1 Cancri	8 16 29·477	·486	...	...	− 9	...	...
η Cancri	8 25 46·042	·105	...	...	−63	...	...
γ Cancri	8 36 20·411	·420	...	...	− 9	...	...
ε Hydræ	8 40 25·214	·247	·230	...	−33	−16	...
α Cancri	8 51 55·415	·395	·382	...	+20	+33	...
κ Cancri	9 1 14·774	·827	...	...	−53	...	...
83 Cancri	9 12 16·940	·965	...	...	−25	...	...
α Hydræ	9 21 41·398	·433	·432	·410	−35	−34	−12
ξ Leonis	9 25 28·588	·594	...	...	− 6	...	...
θ Leonis	9 34 44 690	·709	·725	...	−19	−35	...
ε Leonis	9 39 2·269	·273	·278	...	− 4	− 9	...

Name of Star.	Right Ascensions for 1880·0. Ten-Year Cat.	(*a*)	(*b*)	(*c*)	App. Corrections. (*a*)	(*b*)	(*c*)
	h m s	s					
μ Leonis	9 45 56·170	·211	·206	...	−41	−36	...
π Leonis	9 53 52·267	·282	...	...	−15	...	...
Regulus	10 1 58·798	·818	·804	...	−20	− 6	...
γ^1 Leonis	10 13 21·284	·315	...	...	−31	...	...
μ Hydræ	10 20 17 196	...	...	·234	...	...	−38
ρ Leonis	10 26 29·475	·538	·522	...	−63	−47	...
34 Sextantis	10 36 25·649	·652	...	...	− 3	...	...
l Leonis	10 42 56·905	·956	...	...	−51	...	...
d Leonis	10 54 21·760	·759	...	...	+ 1	...	...
χ Leonis	10 58 49·610	·596	...	...	+14	...	...
δ Leonis	11 7 43·496	·513	·513	...	−17	−17	...
δ Crateris	11 13 20·488	·534	...	·504	−46	...	−16
τ Leonis	11 21 45·949	·952	...	...	− 3	...	...
υ Leonis	11 30 48·259	·290	...	...	−31	...	...
β Leonis	11 42 56·273	·288	·278	...	−15	− 5	...
β Virginis	11 44 26·668	·661	·646	·652	+ 7	+22	+16
π Virginis	11 54 43·388	·392	...	...	− 4	...	...
o Virginis	11 59 5·747	·762	·784	...	−15	−37	...
ϵ Corvi	12 3 57·245	...	...	·288	...	...	−43
η Virginis	12 13 45·967	·015	·996	·996	−48	−29	−29
δ^2 Corvi	12 23 39·354	...	...	·376	...	...	−22
β Corvi	12 28 5·131	·139	...	·126	− 8	...	+ 5
ρ Virginis	12 35 48·629	...	...	...	...	...	...
35 Virginis	12 41 44·799	...	...	...	...	...	...
31 Comæ	12 45 51·153	...	...	...	...	...	...
δ Virginis	12 49 33·524	...	·524	·520	...	0	+ 4
ϵ Virginis	12 56 12·174	...	·208	...	...	−34	...
θ Virginis	13 3 44·247	·242	...	·244	+ 5	...	+ 3
Spica	13 18 52·296	·334	...	·348	−38	...	−52
ζ Virginis	13 28 34·719	·747	·734	·729	−28	−15	−10
m Virginis	13 35 18·844	·889	...	·890	−45	...	−46
τ Bootis	13 41 33·563	...	·594	...	...	−31	...
η Bootis	13 48 58·230	·270	·266	...	...	−36	...
τ Virginis	13 55 32·364	...	·386	·391	...	−22	−27
94 Virginis	13 59 56·570	·533	...	...	+37	...	...
κ Virginis	14 6 29·706	·754	·745	·723	−48	−39	−17
Arcturus	14 10 11·285	·306	·296	...	−21	−11	...
f Bootis	14 20 52·459	...	...	...	...	...	...

Name of Star.	Right Ascensions for 1880·0. Ten-Year Cat.	(a)	(b)	(c)	App. Corrections. (a)	(b)	(c)
	h m s	s					
ρ Bootis	14 26 39·478	·546	·506	...	−68	−28	...
ϵ^2 Bootis	14 39 44·758	·829	...	...	−71	...	...
α Libræ	14 44 14·470	·474	...	·480	− 4	...	−10
ξ^2 Libræ	14 50 15·460	·488	...	·476	−28	...	−16
ψ Bootis	14 59 18·194	...	...	...	...	...	...
ι^1 Libræ	15 5 22·945	...	...	·948	...	...	− 3
β Libræ	15 10 33·010	·041	·038	·028	−31	−28	−18
o^2 Libræ	15 16 20·262	·269	...	...	− 7	...	...
ζ^1 Libræ	15 21 29·407	·422	...	·408	−15	...	− 1
α Coronæ	15 29 36·415	·463	·450	...	−48	−35	...
α Serpentis	15 38 21·424	·473	·462	...	−49	−38	...
ε Serpentis	15 44 50·053	·101	·068	·080	−48	−15	−27
γ Serpentis	15 50 54·622	...	·638	...	...	−16	...
β^1 Scorpii	15 58 27·629	·670	...	·630	−41	...	− 1
δ Ophiuchi	16 8 3·422	·470	·455	·454	−48	−33	−32
γ Herculis	16 16 37·590	...	·590	...	...	0	...
Antares	16 22 3·021	·072	...	·067	−51	...	−46
λ Ophiuchi	16 24 51·671	...	·698	...	...	−27	...
ζ Ophiuchi	16 30 33·074	·116	...	·090	−42	...	−16
ζ Herculis	16 36 45·735	...	·803	...	...	−68	...
κ Ophiuchi	16 51 59·282	·332	·301	...	−50	−19	...
ε Herculis	16 55 41·902	...	·928	...	...	−26	...
η Ophiuchi	17 3 29·781	·789	...	·787	− 8	...	− 6
α^1 Herculis	17 9 10·533	·572	·562	...	−39	−29	..
θ Ophiuchi	17 14 38·432	·433	...	...	− 1	...	...
σ Ophiuchi	17 20 33·631	...	...	...	...	...	...
α Ophiuchi	17 29 21·844	·872	·862	...	−28	−18	...
β Ophiuchi	17 37 32·661	...	·674	·682	...	−13	−21
μ Herculis	17 41 45·700	·777	·774	...	−77	−74	...
89 Herculis	17 50 34·739	...	...	...	...	...	...
72 Ophiuchi	18 1 39·623	...	·633	...	...	−10	...
μ Sagittarii	18 6 35·189	·232	...	·224	−43	...	−35
η Serpentis	18 15 6·023	·053	·030	·052	−30	− 7	−29
λ Sagittarii	18 20 33·875	·892	...	...	−17	...	...
α Lyræ	18 32 52·501	·553	·548	...	−52	−47	...
2 Aquilæ	18 35 42·171	...	...	...	...	...	...
β^2 Lyræ	18 45 38·958	·996	·992	...	−38	−34	...
ε Aquilæ	18 54 10·542	...	·589	...	...	−47	...

Name of Star.	Right Ascensions for 1880·0. Ten-Year Cat.	(a)	(b)	(c)	App. Corrections. (a)	(b)	(c)
	h m s	s					
ζ Aquilæ	18 59 53·667	·696	·676	...	−29	− 9	...
ψ Sagittarii	19 8 10·883	·904	...	...	−21	...	...
ω Aquilæ	19 12 11·008	...	...	...	...	...	...
δ Aquilæ	19 19 26·837	·875	·874	·868	−38	−37	−31
α Vulpec.	19 23 42·712	...	...	...	...	...	...
μ Aquilæ	19 28 13·599	...	...	...	...	...	...
h^2 Sagittarii	19 29 24·222	·218	...	·239	+ 4	...	−17
e^1 Sagittarii	19 33 50·880	·861	...	...	+19	...	...
γ Aquilæ	19 40 33·255	·287	·274	...	−32	−19	...
α Aquilæ	19 44 55·671	·710	·697	...	−39	−26	...
β Aquilæ	19 49 25·072	·131	·114	...	−59	−42	...
c Sagittarii	19 55 16·657	·684	...	...	−27	...	...
θ Aquilæ	20 5 6·752	...	·758	·776	...	− 6	−24
α^2 Capricor.	20 11 23·744	·756	...	·754	−12	...	−10
β Capricor.	20 14 16·093	·105	...	·074	−12	...	+19
ρ Capricor.	20 22 0·859	·909	...	·868	−50	...	− 9
ε Delphini	20 27 28·774	·819	·798	...	−45	−24	...
α Delphini	20 34 3·844	...	·857	...	...	−13	...
ε Aquarii	20 41 10·755	...	·759	·762	...	− 4	− 7
μ Aquarii	20 46 10·808	·853	...	...	−45	...	...
32 Vulpec.	20 49 26·747	...	...	...	...	...	...
θ Capricor.	20 59 11·986	·073	...	...	−87	...	...
ζ Cygni	21 7 49·762	·751	·760	...	+11	+ 2	...
α Equulei	21 9 49·486	...	·496	·498	...	−10	−12
ι Capricor.	21 15 33·817	·819	...	...	− 2	...	...
β Aquarii	21 25 14·435	·479	·470	·466	−44	−35	−31
ξ Aquarii	21 31 21·774	·809	...	...	−35	...	...
ε Pegasi	21 38 17·512	·558	·544	...	−46	−32	...
δ Capricor.	21 40 24·968	·025	...	...	−57	...	...
16 Pegasi	21 47 36·136	...	...	...	...	...	...
α Aquarii	21 59 37·199	·218	·205	·220	−19	− 6	−21
ι Pegasi	22 1 25·506	...	·521	...	...	−15	...
θ Aquarii	22 10 30·035	·056	...	·047	−21	...	−12
γ Aquarii	22 15 27·462	...	·472	·478	...	−10	−16
σ Aquarii	22 24 17·755	·730	...	·762	+25	...	− 7
η Aquarii	22 29 11·366	·394	·387	·390	−28	−21	−24
ζ Pegasi	22 35 28·632	·659	·651	...	−27	−19	...
μ Pegasi	22 44 12·730	...	·746	...	...	−16	...

Name of Star.	Right Ascensions for 1880·0. Ten-Year Cat.	(*a*)	(*b*)	(*c*)	App. Corrections. (*a*)	(*b*)	(*c*)
	h m s	s					
λ Aquarii	22 46 21·205	·240	·204	·208	−35	+ 1	− 3
Fomalhaut	22 51 1·005	·024	...	...	−19	...	...
α Pegasi	22 58 47·025	·048	·032	...	−23	− 7	...
γ Piscium	23 10 56·660	...	·661	·656	...	− 1	+ 4
κ Piscium	23 20 46·842	·852	...	·870	−10	...	−28
ι Piscium	23 33 46·670	·715	·702	·694	−45	−32	−24
δ Sculptor.	23 42 40·401	...	...	...	...	...	...
ω Piscium	23 53 8·938	·990	·966	...	−52	−28	...
2 Ceti	23 57 35·469	...	...	·504	...	...	−35

The systematic differences among the catalogues are shown by taking the mean of the corrections for each quadrant. They are :—

	(*a*)	(*b*)	(*c*)
	s	s	s
Quadrant I.	−·034	−·032	−·022
„ II.	−·030	−·023	−·025
„ III.	−·033	−·027	−·019
„ IV.	−·029	−·019	−·017
Mean	−·032	−·025	−·021

The near approach to equality among the numbers of the first column shows that the periodic difference between (*a*) and the results of Greenwich observations between 1840 and 1870 has been eliminated by the system of 12-hour groups. In the case of the Greenwich Six-Year Catalogue for 1840 the difference was

$$+0^{s}\cdot014\cos\alpha - 0^{s}\cdot034\sin\alpha,$$

and it has continually diminished in the successive catalogues which have since appeared.

Quite remarkable is the large difference of equinoctial point between Greenwich and the other catalogues, which I pointed out in the *Monthly Notices*, vol. xxxv. p. 405, and which has now persisted for about half a century. Its source is probably to be sought not in the absolute declinations of the Sun given by observations, but in the difference between the observed times of transit of the Sun and of the stars. It seems to show an *instrumental* personal equation of a kind not hitherto noticed, and possibly due to differences of thickness of transit threads.

Of the list (*a*) 101 stars belong to the fundamental catalogue of the American Ephemeris. From each of the corrections (*a*)

to these stars the constant $0^s{\cdot}037$ was subtracted and the mean of the residuals taken, without regard to sign, with the result—

$$\text{Mean } r = \pm 0^s{\cdot}0145.$$

This corresponds to—

$$\text{Probable difference} = \pm 0^s{\cdot}012.$$

Had this determination been extended to all the stars in the list the probable difference would doubtless be found larger.

Comparison of the Greenwich Ten-Year Catalogue with the Williamstown Right Ascensions of Polar Stars for 1885. By Professor Truman Henry Safford.

It is pretty sure that the personal equation of an observer varies in some degree with the apparent velocity of the star. Hence different observers will, as a rule, assign different right ascensions to slow-moving stars if they determine them, as is usual, by comparison with quick-moving ones. The investigation of this matter can be facilitated by a study of the right ascensions of high northern stars as given in various catalogues. While we cannot say that the systematic differences near the pole are altogether dependent upon personal equation, we can at least infer that this is partly the case.

The *Greenwich Ten-Year Catalogue* for 1880, just published, contains 106 stars in common with the *Williamstown Catalogue of Polar Right Ascensions* for 1885; and the interval of time between them—on the average no more than three or four years—enables a comparison to be made without much uncertainty, due to proper motion.

Such a comparison I have made, employing Professor Auwers's proper motions as given in the Greenwich Catalogue (except in isolated cases, where other values are decidedly better), or, lacking these, proper motions determined by Argelander or by myself from a least-square discussion of all available material.

The primary object of the comparison, together with many others of the same kind which I have made, has been to obtain the means of deducing final corrections to be applied in perfecting a least-square discussion, which I have nearly completed, of 170 stars within 10° of the North Pole.

The following are the results of the comparison, in zones of convenient width. The Williamstown Catalogue contains no stars south of +65°, and only selected stars between +65° and +80°; but between +80° and the pole is nearly complete as far as the seventh magnitude inclusive:—

Declination.	No. of Stars.	Mean $\Delta\alpha \cos\delta$. W.—G.	Probable Error of 1 Star.	Probable Error of Mean.
° °		s	s	s
86–90	12	+0·0257	±0·0249	±0·0063
84–86	10	+0·0379	±0·0203	±0·0070
82–84	13	+0·0266	±0·0221	±0·0061
80–82	24	+0·0343	±0·0196	±0·0045
77–80	13	+0·0288	±0·0215	±0·0061
74–77	12	+0·0158	±0·0186	±0·0063
70–74	11	+0·0319	±0·0304	±0·0066
65–70	11	+0·0211	±0·0218	±0·0066

The probable errors for one star are derived from the sums of the errors, as compared with the mean of the groups, by the usual formula; their average $\pm 0^s\cdot022$ is taken to calculate the probable errors of the means; and it will be at once seen that these separate means differ among themselves less, on the average, than the probable errors would require; so that there is no reason why we should not accept the general mean—

$$\Delta\alpha \cos\delta = 0^s\cdot0283$$

from all the stars as perfectly representing these separate groups.

In other words, the Greenwich right ascensions of polar stars for 1880 are less than the Williamstown right ascensions for 1885 by $0^s\cdot0283$ sec. δ.

But, on the other hand, when the stars are arranged according to right ascension, we find an apparent slight tendency to periodicity.

I have divided them into seven groups, as follows:—

Limits of R.A. 1880·0.				Mean $\Delta\alpha \cos\delta$.	No. of Stars.	P.E. 1 Star.	P.E. of Mean.
h	m	h	m	s		s	s
0	9	4	2	+0·0369	15	±0·0150	±0·0052
4	3	7	35	+0·0391	15	±0·0170	±0·0052
7	36	12	48	+0·0264	15	±0·0264	±0·0052
13	23	15	39	+0·0206	15	±0·0178	±0·0052
15	45	18	22	+0·0319	15	±0·0297	±0·0052
18	35	21	22	+0·0049	15	±0·0178	±0·0052
21	28	23	54	+0·0381	16	±0·0185	±0·0051

The separate groups here differ from the general mean $+0^s\cdot0283$, by quantities of the same order as the probable error of the single means, except in the sixth group, $18^h\ 35^m$–$21^h\ 22^m$,

where the discrepancy is rather more than four times its probable error.

The attempt was made, by comparison between Greenwich and Cambridge, 1875 (Professor Rogers's Catalogue of 1213 stars), to come to a more distinct conclusion on this matter; but the only result was to exhibit a somewhat larger and more extended periodicity in the differences for polar stars, so that I must defer further study on this point till I have accumulated a larger stock of materials.

Williams' College Observatory:
1890 *May* 29.

The Star-Places of the Second Melbourne General Catalogue for 1880. By A. M. W. Downing, M.A.

For the purpose of discussing the star-places of the Second Melbourne General Catalogue for 1880, they have been compared with those of the Cape Catalogue for the same epoch. In some cases different values of the proper motions have been adopted in reducing the observations from the mean dates of observation to the common epoch of the two catalogues. It has, however, been assumed that the effect of the adopted proper motions would not sensibly affect the comparison, when such a large number of stars are available, as is the case in the present instance. Every star common to the catalogues, and which has been observed in both elements in each, has been used in the comparison. Only one *erratum* has been detected during the progress of the work—viz. Melbourne No. 43, for N.P.D. 83° 3′ 5″·27 read 83° 4′ 5″·27.

Table I., given below, gives the differences Cape—Melbourne when the stars are arranged in order of N.P.D., and grouped as shown in the table. The columns "curve" give the readings from curves drawn through the several differences, when laid down as points on cross-ruled paper. It will be seen that the mean differences in R.A. and N.P.D. are $+0^{s}\cdot013$ and $+0''\cdot61$ respectively; and that the total number of stars used in the comparison is 881.

Table II. gives the same details as Table I., the stars in this instance, however, being arranged in order of R.A., and the mean differences having been applied, with reversed signs, to the hourly means before their entry in this table.

Tables III. and IV. are formed by reading off from the curves mentioned above the quantities corresponding to the beginning of each hour of R.A., and each 5° of N.P.D.

By the aid of the tables printed on pages 453 and 454 of

vol. xlvii. of the *Monthly Notices*, the comparison is extended so as to include the Cordoba and First Melbourne General Catalogue. In the formation of Tables III. and IV. it must be remarked that the signs of the quantities shown have been arranged so as to give corrections applicable to the star-places of the Cape, Cordoba, and First Melbourne Catalogues, in order to reduce them respectively to the system of the Second Melbourne General Catalogue for 1880.

Table I.

N.P.D.	Δα Means.	Δα Curve.	Δδ Means.	Δδ Curve.	No. of Stars in group.
° °	s	s	″	″	
40–50	−0·016	...	−0·85	...	5
50–60	+ ·013	+0·003	−0·71	−0·59	7
60–65	+ ·001	+ ·005	−0·09	− ·17	17
65–70	+ ·006	+ ·006	+0·22	+ ·25	22
70–75	+ ·008	+ ·005	+0·62	+ ·40	14
75–80	·000	− ·002	+0·12	+ ·24	21
80–85	− ·016	− ·012	+0·07	+ ·13	27
85–90	− ·013	− ·007	+0·25	+ ·27	25
90–95	+ ·014	+ ·003	+0·48	+ ·53	44
95–100	− ·005	+ ·003	+0·89	+ ·80	42
100–105	+ ·006	+ ·009	+0·91	+ ·85	52
105–110	+ ·026	+ ·019	+0·68	+ ·75	44
110–115	+ ·017	+ ·017	+0·72	+ ·67	38
115–120	+ ·004	+ ·008	+0·56	+ ·64	20
120–125	+ ·005	+ ·017	+0·71	+ ·62	90
125–130	+ ·050	+ ·036	+0·48	+ ·62	78
130–135	+ ·036	+ ·039	+0·78	+ ·71	100
135–140	+ ·031	+ ·037	+0·79	+ ·78	73
140–145	+ ·047	+ ·052	+0·75	+ ·79	17
145–150	+ ·081	+ ·068	+0·86	+ ·79	20
150–155	+ ·061	+ ·040	+0·70	+ ·70	14
155–160	− ·044	− ·012	+0·52	+ ·70	15
160–165	− ·019	− ·053	+1·04	+ ·86	8
165–170	− ·127	− ·084	+0·84	+ ·92	17
170–175	− ·063	− ·056	+0·95	+ ·73	41
175–180	+ ·024	...	+0·16	...	30
40–180	+0·013	...	+0·61	...	881

Table II.

R.A.	Δα Means.	Δα Curve.	Δδ Means.	Δδ Curve.	No. of Stars in group.
h h	s	s	″	″	
0–1	+0·030	+0·020	−0·06	+0·20	51
1–2	− ·004	+ ·014	+ ·40	+ ·20	50
2–3	+ ·031	+ ·010	+ ·06	+ ·07	46
3–4	− ·021	− ·007	− ·25	− ·04	40
4–5	− ·015	− ·017	+ ·30	+ ·16	38
5–6	− ·014	− ·013	+ ·25	+ ·21	49
6–7	− ·008	− ·014	+ ·03	+ ·07	40
7–8	− ·026	− ·015	− ·05	+ ·04	40
8–9	+ ·003	− ·002	+ ·22	+ ·08	49
9–10	+ ·012	+ ·015	− ·08	− ·02	41
10–11	+ ·030	+ ·033	− ·14	− ·13	41
11–12	+ ·058	+ ·047	− ·16	− ·22	36
12–13	+ ·039	+ ·035	− ·42	− ·33	32
13–14	+ ·001	+ ·032	− ·31	− ·39	25
14–15	+ ·084	+ ·036	− ·49	− ·43	24
15–16	− ·028	+ ·015	− ·48	− ·39	22
16–17	+ ·030	+ ·005	− ·17	− ·35	25
17–18	− ·012	− ·006	− ·55	− ·35	27
18–19	− ·029	− ·015	− ·10	− ·30	31
19–20	+ ·014	− ·005	− ·41	− ·25	26
20–21	− ·016	− ·025	− ·07	− ·24	27
21–22	− ·081	− ·042	− ·39	− ·16	34
22–23	+ ·013	− ·008	+ ·24	+ ·15	38
23–24	+ ·023	+ ·023	+ ·50	+ ·30	49

Table III.

R.A.	Cape, 1880.		Cordoba, 1875.		Melbourne, 1870.	
h	s	″	s	″	s	″
0·0	−0·021	−0·25	+0·003	−0·06	+0·012	−0·02
1·0	·017	·20	·010	·06	+ ·011	− ·08
2·0	·011	·14	·005	− ·04	− ·015	·00
3·0	− ·003	·01	·007	+ ·01	·046	+ ·22
4·0	+ ·013	·04	·033	− ·11	·032	·23
5·0	·016	·20	·026	·27	− ·001	+ ·03
6·0	·014	·15	+ ·001	·27	+ ·008	− ·09
7·0	·016	·04	− ·016	·19	− ·008	·09
8·0	+ ·010	·05	·020	·16	·036	− ·04
9·0	− ·007	− ·03	·021	− ·07	·054	+ ·06
10·0	·024	+ ·08	·018	+ ·07	·059	− ·02
11·0	·042	·18	·038	·12	·057	·14

R.A.	Cape, 1880.		Cordoba, 1875.		Melbourne, 1870.	
h	s	″	s	″	s	″
12·0	−0·041	+0·28	−0·074	+0·22	−0·038	−0·12
13·0	·032	·36	·072	·33	·015	− ·06
14·0	·032	·42	·057	·35	− ·009	+ ·16
15·0	·024	·43	·033	·29	+ ·012	·28
16·0	− ·010	·37	− ·011	·23	·032	·24
17·0	·000	·35	+ ·006	·26	·029	·27
18·0	+ ·012	·32	·023	·36	·022	·26
19·0	·010	·27	·023	·39	·010	·28
20·0	·015	·25	·034	·39	·017	·35
21·0	·033	·21	·058	·44	·052	·37
22·0	+ ·028	+ ·03	·053	·30	·060	·23
23·0	− ·010	− ·23	+ ·010	+ ·02	+ ·021	+ ·02

TABLE IV.

N.P.D.	Cape, 1880.		Cordoba, 1875.		Melbourne, 1870.	
°	s	″	s	″	s	″
55	−0·003	+0·59	...	...	−0·010	−0·31
60	·004	+ ·33	...	...	·006	·32
65	·005	− ·02	...	...	·002	·33
70	·005	·35	...	...	− ·001	·34
75	− ·002	·31	...	...	+ ·011	·09
80	+ ·008	·17	...	...	·025	·25
85	·011	·18	...	...	·020	·41
90	+ ·002	·39	...	...	+ ·002	·49
95	− ·004	·67	−0·032	+0·05	− ·009	·52
100	·006	·85	·038	− ·14	·020	·50
105	·013	·80	·051	·10	·038	·31
110	·019	·70	·060	·33	·056	·26
115	·011	·65	·059	·46	·049	·35
120	·010	·63	·074	·31	·046	·44
125	·026	·63	·091	·31	·051	·48
130	·038	·67	·098	·47	·052	·37
135	·038	·76	·116	·44	·064	·26
140	·044	·78	·132	·28	·077	·26
145	·060	·79	·108	·26	·073	·25
150	·053	·75	·063	·28	·058	− ·11
155	− ·015	·70	− ·044	·28	·039	+ ·03
160	+ ·032	·77	·000	·33	·037	·07
165	·070	·90	+ ·077	·10	−·013	+ ·03
170	·070	·87	·128	·04	+·003	− ·04
175	+ ·041	− ·70	+ ·154	− ·23	−·022	− ·10

Corrections to the Elements of the Orbit of Juno.

By A. M. W. Downing, M.A.

My attention having been drawn to the large and increasing errors of the computed places of *Juno*, as given in the *Nautical Almanac* (amounting to some 3′ in geocentric longitude in 1887), I have endeavoured in the present paper to deduce corrections to the assumed elements of the orbit, so as to obtain elements which shall represent the observed places of the planet with sufficient accuracy for, I hope, some years to come.

The observations of *Juno* which are discussed in this paper are those published in the volumes of *Greenwich Observations* from 1864 to 1887 inclusive. The discussion therefore embraces observations made at the Paris Observatory during the years 1864 to 1878, as well as those made at Greenwich during the whole period. Le Verrier's Tables of the Sun were first used in the *Nautical Almanac* for the year 1864, and it has been assumed that the tabular places of the Sun are sensibly accurate throughout the years with which we are here concerned.

The first step in the discussion of the observations, both Greenwich and Paris, is to reduce them to a uniform system. That adopted for the R.A.'s is the standard system of R.A.'s deduced from 12-hour groups of clock-stars for 1872·0, printed on pp. 21 and 22 of the Introduction to the *Greenwich Nine-Year Catalogue*, and which has been used in the reduction of the Greenwich observations for 1878 to 1888 inclusive. For the years 1864 to 1869 the Greenwich R.A.'s depend on those of the First Seven-Year Catalogue, whilst for the years 1872 to 1877 they depend on those of the Second Seven-Year Catalogue. The following table (deduced from the comparisons printed on page 8 of the above-mentioned Introduction) gives the corrections applicable to the R.A.'s observed at Greenwich during the specified periods to reduce them to the adopted system:—

R.A.	1864—1869.	1870—1877.
h h	s	s
0–3	+0·02	+0·02
3–6	− ·01	+ ·01
6–9	− ·02	·00
9–12	− ·01	+ ·01
12–15	·00	+ ·01
15–18	·00	+ ·01
18–21	+ ·02	+ ·03
21–24	+ ·03	+ ·03

Observations made since 1878·0 require no correction.

In the reduction of the Paris observations of R.A. a "provisional" standard catalogue (constructed in 1854) was used

during the period 1864–1871, with different corrections each year, deduced from then recent observations; whilst during the period 1872–1878 the corrections to the provisional catalogue were deduced from observations extending over the years 1856–1869. It was found, however, that the corrections given in the following table would reduce the Paris observations made throughout the whole period 1864–1878 to the adopted system with sensible accuracy:—

R.A.	Corr. to Paris.	R.A.	Corr. to Paris.
h h	s	h h	s
0–3	+0·02	12–15	+0·01
3–6	− ·01	15–18	+ ·02
6–9	− ·01	18–21	+ ·03
9–12	·00	21–24	+ ·03

With regard to the N.P.D.'s observed at Greenwich during the years 1864–1882, it is necessary to correct them for the law for R.–D.—viz. $a+b'$ sin z cos ^2z, assumed during those years, so as to make them depend on the law $a+b$ sin z. This has been done by assuming that the mean Z.D. at which reflection observations were made during those years (and from which, therefore, the values of a and b' were deduced) was 25°, and that therefore $b=\cdot 8\times b'$. It is also necessary to correct observations made during 1864–1876 by the quantities given in the table on p. 27 of the Addendum to the Introduction to the *Nine-Year Catalogue*, to reduce them to the refractions of Bessel's Tabulæ, and to colatitude 38° 31′ 21″·90. It is further necessary to correct observations made during 1868–1874, for errors due to wear in the micrometer-screws of the transit-circle, by the quantities given on p. 14 of the Introduction to the *Nine-Year Catalogue*. The following table gives the corrections to the observed N.P.D.'s corresponding to the N.P.D. of *Juno*, at the time of opposition, in the different years (exclusive of the correction for wear of screws, which was applied to each individual observation):—

Year.	N.P.D.	Corr.	Year.	N.P.D.	Corr.
	°	″		°	″
1864	94	−0·3	1874	94	+1·2
1865	94	−0·7	1876	87	+1·0
1867	86	+0·3	1877	94	+0·7
1868	92	+1·3	1878	94	+0·6
1869	95	+1·2	1880	86	+0·3
1871	87	+1·0	1881	92	+0·1
1872	90	+1·1	1882	95	+0·2
1873	95	+1·3			

Observations made since 1883·0 require no correction.

The Paris observations in N.P.D. depend on the assumed places of standard stars taken from the corrected "provisional" catalogue referred to above. From a discussion of the places of stars in this catalogue, situated between N.P.D. 70° and 110°, it was found that the following corrections were applicable to Paris observations, between these limits of N.P.D., made during the period 1864–1878, to reduce them to the adopted system of N.P.D.'s:—

R.A. h h	Corr. to Paris. ″	R.A. h h	Corr. to Paris. ″
0–3	+0·4	12–15	+0·4
3–6	+0·3	15–18	+0·6
6–9	+0·2	18–21	+0·7
9–12	+0·2	21–24	+0·5

The publication of Paris observations of minor planets in the volumes of *Greenwich Observations* was intermitted after 1879; and there are no Paris observations of *Juno* printed in the Greenwich volumes after 1878.

The requisite corrections, deduced from those given in the foregoing tables, have been applied to the quantities given in the section "Errors of the Tabular Heliocentric Places of the Planets," printed in successive volumes of *Greenwich Observations*. The corrected equations connecting the error of tabular heliocentric longitude (δl), the error of tabular heliocentric ecliptic north polar distance (δb), and the error of projected radius vector of *Juno* ($\delta\rho$) are given below, together with the mean dates of the groups from which the normal places have been formed, and the number of observations in each group.

No.	Mean Date.	*Equations.*		No. of Obs.
1	1864 June 10	$-\ 8''\cdot44 = +1\cdot462\delta l'' + 3102''\delta\rho,$	$\delta b'' = +\ 0''\cdot04 - 6546''\delta\rho$	5
2	25	$-\ 8\cdot75 = +1\cdot418 + 11728$	$= -\ 0\cdot40 - 6294$	5
3	July 6	$-\ 6\cdot84 = +1\cdot360 + 16835$	$= -\ 0\cdot16 - 5862$	2
4	1865 Sept. 2	$-\ 27\cdot31 = +1\cdot769 - 9819$	$= -\ 3\cdot98 - 1756$	4
5	13	$-\ 27\cdot10 = +1\cdot788 + 6490$	$= -\ 4\cdot75 - 892$	8
6	27	$-\ 25\cdot83 = +1\cdot729 + 27421$	$= -\ 4\cdot35 + 335$	8
7	1867 Mar. 1	$-\ 22\cdot47 = +1\cdot574 - 3316$	$= +\ 4\cdot71 + 1354$	5
8	27	$-\ 22\ 57 = +1\cdot477 + 18293$	$= +\ 3\cdot70 + 375$	5
9	1868 May 7	$-\ 16\cdot89 = +1\cdot436 - 3383$	$= +\ 1\cdot93 - 5377$	1
10	17	$-\ 15\cdot70 = +1\cdot441 + 2288$	$= +\ 1\cdot97 - 5480$	4
11	25	$-\ 16\cdot56 = +1\cdot426 + 6730$	$= +\ 1\cdot21 - 5442$	4
12	June 6	$-\ 16\cdot32 = +1\cdot379 + 12640$	$= +\ 1\ 36 - 5186$	4
13	1869 July 17	$-\ 31\cdot16 = +1\cdot573 - 8243$	$= -\ 3\ 66 - 7444$	4
14	29	$-\ 31\cdot73 = +1\cdot594 + 4905$	$= -\ 3\cdot18 - 7439$	4
15	Aug. 5	$-\ 30\cdot45 = +1\cdot583 + 9421$	$= -\ 3\cdot53 - 7283$	3

No.	Mean Date.	*Equations.*			No. of Obs.
16	1869 Aug. 13	$-29''.49 = +1.549\delta l'' + 16597''\delta\rho,$	$\delta b''$	$= -4''.49 - 6974''\delta\rho$	3
17	1871 Jan. 26	$-59.94 = +1.787 + 9813$		$= +7.89 + 12476$	1
18	Feb. 13	$-60.16 = +1.633 + 32398$		$= +9.47 + 10027$	1
19	1872 Apr. 19	$-26.53 = +1.450 - 1293$		$= +3.32 - 4220$	3
20	May 4	$-27.55 = +1.426 + 7937$		$= +4.18 - 4272$	5
21	1873 June 23	$-35.61 = +1.500 - 1686$		$= -1.49 - 7297$	5
22	July 17	$-32.43 = +1.446 + 14526$		$= -0.94 - 6938$	4
23	1874 Nov. 12	$-132.25 = +2.004 + 17358$		$= -10.79 + 19646$	5
24	1876 Mar. 30	$-43.45 = +1.492 + 2164$		$= +7.65 - 2284$	6
25	Apr. 19	$-41.45 = +1.420 + 14570$		$= +7.67 - 2445$	5
26	1877 June 4	$-41.02 = +1.457 + 2059$		$= +1.15 - 6338$	4
27	22	$-38.80 = +1.406 + 12450$		$= +0.88 - 6049$	4
28	1878 Aug. 23	$-93.16 = +1.722 - 7613$		$= -11.79 - 4033$	2
29	Sept. 5	$-94.60 = +1.729 + 9874$		$= -13.78 - 3296$	4
30	21	$-90.04 = +1.637 + 29940$		$= -13.69 - 2138$	7
31	1880 Feb. 18	$-72.86 = +1.609 - 6716$		$= +11.69 + 3200$	1
32	Mar. 19	$-71.42 = +1.496 + 21244$		$= +12.49 + 1676$	6
33	1881 May 12	$-45.20 = +1.443 + 2110$		$= +4.46 - 5276$	4
34	June 1	$-45.64 = +1.382 + 12562$		$= +4.77 - 5011$	4
35	1882 July 18	$-71.79 = +1.567 - 368$		$= -5.53 - 7546$	3
36	Aug. 13	$-69.50 = +1.481 + 20906$		$= -7.46 - 6794$	5
37	Sept. 18	$-55.07 = +1.188 + 33701$		$= -10.19 - 4053$	5
38	1884 Jan. 11	$-162.40 = +1.868 + 12076$		$= +11.98 + 17185$	1
39	Feb. 17	$-123.40 = +1.448 + 49724$		$= +14.08 + 10654$	9
40	Mar. 14	$-97.10 = +1.169 + 48987$		$= +14.18 + 5971$	2
41	1885 Mar. 30	$-57.77 = +1.426 - 9646$		$= +10.97 - 3541$	1
42	Apr. 20	$-56.92 = +1.457 + 2821$		$= +8.18 - 3959$	1
43	May 2	$-56.92 = +1.424 + 9740$		$= +9.03 - 3944$	1
44	19	$-51.90 = +1.332 + 17066$		$= +7.82 - 3628$	2
45	1886 June 11	$-63.79 = +1.476 - 5791$		$= -0.90 - 6972$	2
46	July 1	$-63.44 = +1.474 + 7829$		$= -1.39 - 7016$	5
47	14	$-63.69 = +1.418 + 15516$		$= -2.38 - 6617$	3
48	1887 Oct. 14	$-214.36 = +1.958 - 3690$		$= -23.42 + 11468$	5
49	22	$-213.82 = +1.954 + 13584$		$= -23.48 + 12421$	5
50	31	$-210.09 = +1.901 + 31203$		$= -23.44 + 13446$	2
51	Nov. 16	$-191.41 = +1.714 + 56008$		$= -21.07 + 14225$	2
52	28	$-173.55 = +1.542 + 66540$		$= -19.74 + 14028$	3
53	Dec. 7	$-159.18 = +1.420 + 69854$		$= -18.43 + 13511$	2
54	19	$-147.48 = +1.266 + 71960$		$= -16.40 + 12388$	2

The next step is to eliminate $\delta\rho$ between the two equations corresponding to each normal date; so that we have a single equation of the form

$$\alpha\delta l + \beta\delta b + \gamma = 0,$$

corresponding to each normal date.

To express the variations of heliocentric longitude and heliocentric ecliptic north polar distance in terms of the variations of the elements of orbit, the following formulæ have been used :—

$$\begin{aligned}
\delta l = {} & \frac{\cos i\, a^2}{\cos^2 \mathrm{B}\, \mathrm{R}^2} \cos\phi \{(t-t_0)\delta n + \delta\epsilon\} \\
& + \frac{\cos i}{\cos^2 \mathrm{B}} \left(1 - \frac{a^2}{\mathrm{R}^2}\cos\phi\right) \delta\pi \\
& + \left(1 - \frac{\cos i}{\cos^2 \mathrm{B}}\right) \delta\Omega \\
& - \tan \mathrm{B} \cos(l-\Omega)\, \delta i \\
& + \frac{\cos i}{\cos^2 \mathrm{B}} \left(\frac{2}{\cos\phi} + \tan\phi\cos v\right) \sin v\, \delta\phi, \\
-\delta b = {} & \sin i \cos(l-\Omega) \frac{a^2}{\mathrm{R}^2} \cos\phi \{(t-t_0)\delta n + \delta\epsilon\} \\
& + \sin i \cos\left(l-\Omega\right)\left(1 - \frac{a^2}{\mathrm{R}^2}\cos\phi\right) \delta\pi \\
& - \sin i \cos(l-\Omega)\, \delta\Omega \\
& + \sin(l-\Omega)\, \delta i \\
& + \sin i \cos(l-\Omega) \left\{\frac{2}{\cos\phi} + \tan\phi\cos v\right\} \sin v\, \delta\phi.
\end{aligned}$$

where

l is the heliocentric longitude,
B „ „ latitude,
b „ „ ecliptic north polar distance,
a „ semi-axis major of orbit,
R „ radius vector of planet,
v „ true anomaly of planet,
$t-t_0$ „ time in days from 1876, March $19^{d}\cdot0$

and

n, ϵ, π, Ω, i and e $(=\sin\phi)$ are the elements of the orbit.

The required quantities have been taken from, or have been computed from quantities taken from, the *Nautical Almanacs* of the different years.

By the substitution of the values of δl and δb thus obtained in the equations of the form $\alpha\delta l + \beta\delta b + \gamma = 0$, the following equations of condition (where $\delta\mathrm{N} = 1000\ \delta n$), connecting the errors of the six elements, have been formed. The adopted weight is simply the number of observations on which each equation depends.

These equations of condition have been solved by the method of least squares, and the subjoined normal equations formed; the values of the six unknown quantities derived from which have been substituted in the equations of condition, and the residuals (given in the last column) computed.

It is to be observed that in the equations of condition the coefficients are logarithms, whilst in the normal equations the coefficients are natural numbers.

Equations of Condition.

No.							Weights.	Residuals.
1	$-0.61839\delta N$	$+9.98498\delta\epsilon$	$+9.73320\delta\pi$	$-8.65321\delta\Omega$	$+9.64542\delta i$	$-9.99651\delta\phi$	$+ 8''.46 = 0$	5	$+ 0''.64$
2	-0.61209	$+9.98000$	$+9.71600$	-8.76343	$+0.25600$	-0.02857	$+ 8.00 = 0$	5	$+ 0.56$
3	-0.59415	$+9.96332$	$+9.69108$	-8.69897	$+0.45697$	-9.07188	$+ 6.38 = 0$	2	$- 0.17$
4	-1.17541	$+0.58984$	-9.95999	-0.08243	-9.74974	-0.76864	$+ 49.57 = 0$	4	$+ 1.61$
5	-9.73277	$+9.14922$	-8.55630	$+0.22608$	$+9.60853$	-8.86923	$- 7.46 = 0$	8	$- 2.10$
6	-2.02732	$+1.44458$	-0.88694	-1.26475	$+0.22246$	-1.53596	$+381.90 = 0$	8	$- 0.53$
7	-0.80576	$+0.28646$	$+9.10037$	-9.68753	-9.84323	$+0.61857$	$+ 34.00 = 0$	5	$+ 4.69$
8	$+1.43502$	-0.91913	-9.99782	$+1.03238$	$+1.02094$	-1.26002	$-157.92 = 0$	5	$+ 7.02$
9	-0.39699	$+9.92840$	$+9.73159$	$+8.67210$	-9.84011	$+8.97772$	$+ 15.68 = 0$	1	$+ 1.01$
10	-0.42049	$+9.96332$	$+9.76864$	-8.81291	$+9.37107$	$+8.57978$	$+ 16.52 = 0$	4	$- 0.30$
11	-0.43782	$+9.98227$	$+9.78675$	-9.16435	-9.99078	-8.17609	$+ 18.06 = 0$	4	$+ 0.34$
12	-0.45215	$+9.99826$	$+9.80277$	-9.40140	$+0.32160$	-9.00000	$+ 19.63 = 0$	4	$+ 1.13$
13	-0.57634	$+0.18949$	$+9.29003$	-9.22789	-9.81823	-0.49415	$+ 35.21 = 0$	4	$+ 1.06$
14	-0.51746	$+0.13258$	$+9.13988$	$+8.85733$	$+9.82217$	-0.43791	$+ 29.63 = 0$	4	$+ 0.89$
15	-0.48684	$+0.10346$	$+9.04139$	$+9.30963$	$+0.04218$	-0.40824	$+ 25.88 = 0$	3	$- 0.64$
16	-0.41682	$+0.03503$	$+8.88081$	$+9.59106$	$+0.26007$	-0.33905	$+ 18.80 = 0$	3	$- 2.48$
17	-0.62991	$+0.35603$	-9.77232	$+9.03743$	$+9.90200$	$+0.52673$	$+ 53.73 = 0$	1	$- 0.63$
18	-0.42588	$+0.15594$	-9.51720	$+9.72428$	$+0.38274$	$+0.35353$	$+ 29.56 = 0$	1	$- 2.37$

No.							Weights.	Residuals.
19	$-0.10781\delta N$	$+9.95231\delta\epsilon$	$+9.69461\delta\pi$	$+8.77085\delta\Omega$	$-9.56110\delta i$	$+9.99870\delta\phi$	$+ 25''.51 = 0$	3	$+ 0''.52$
20	-0.19398	$+0.04336$	$+9.79727$	-9.48430	$+0.03902$	$+0.04610$	$+ 35.32 = 0$	5	$+ 3.36$
21	-0.03908	$+0.03902$	$+9.65610$	-8.67210	-9.17319	-0.25575	$+ 35\ 95 = 0$	5	$+ 0.89$
22	-9.97839	$+9.98900$	$+9.56110$	$+9.02531$	$+0.32139$	-0.23019	$+ 30.46 = 0$	4	$+ 0.42$
23	-0.26441	$+0.57148$	-0.20167	-9.12710	$+9.69984$	-9.87795	$+ 141.78 = 0$	5	$+ 12.45$
24	$+8\ 13799$	$+0.09656$	$+9.61909$	-9.23553	$+9.25763$	$+0.35736$	$+ 50.70 = 0$	6	$+ 2.62$
25	$+8.77875$	$+0.28735$	$+9.85370$	-0.09026	$+0.29292$	$+0.53161$	$+ 87.16 = 0$	5	$+ 8.91$
26	$+9.62226$	$+9.97681$	$+9.74663$	-8.68124	$+9.41996$	-9.88252	$+ 41.39 = 0$	4	$+ 1.49$
27	$+9.64175$	$+9.97909$	$+9.73799$	-8.97313	$+0.30146$	-9.94399	$+ 40.61 = 0$	4	$+ 0.75$
28	$+0.37027$	$+0.39533$	-9.59329	-9.56937	-9.62941	-0.63949	$+ 115.42 = 0$	2	$+ 0.80$
29	$+0.05664$	$+0.10209$	-9.36922	$+9.84386$	$+9.87967$	-0.33082	$+ 53.32 = 0$	4	$- 0.72$
30	-0.24977	-0.28780	$+9.62014$	$+0.49969$	$+0.32899$	$+0.49541$	$- 101.67 = 0$	7	$- 0.54$
31	$+0.46994$	$+0.31429$	-8.68124	-9.60853	-9.69020	$+0.60670$	$+ 97.39 = 0$	1	$- 3.28$
32	-0.27996	-0.11528	-8.70757	$+0.45500$	$+0.34537$	-0.41714	$- 86.90 = 0$	6	$- 11.85$
33	$+0.24210$	$+9.96802$	$+9.76268$	-8.80618	$+9.28103$	$+9.44404$	$+ 46.98 = 0$	4	$- 1.50$
34	$+0.29128$	$+0.01242$	$+9.81023$	-9.46687	$+0.31345$	$+9.34439$	$+ 57.60 = 0$	4	$+ 3.47$
35	$+0.49051$	$+0.12646$	$+9.39620$	-8.30103	$+9.12057$	-0.42243	$+ 72.06 = 0$	3	$- 2.25$
36	$+0.32917$	$+9.95999$	$+9.08636$	$+9.65031$	$+0.39777$	-0.26031	$46.53 = 0$	5	$- 1.59$

No.	$-9.677485N$	-9.303206ϵ	-8.079186π	$+0.146136\Omega$	$+0.754046i$	$+9.605316\phi$		Weights.	Residuals.
37							$-29''.66=0$	5	$-10''.82$
38	$+0.89340$	$+0.43791$	-9.95952	$+8.57978$	$+9.90417$	$+0.49762$	$+153.98=0$	1	$+6.36$
39	$+0.51212$	$+0.05115$	-9.49554	$+9.80346$	$+0.58501$	$+0.19700$	$+57.69=0$	9	-4.15
40	-9.77789	-9.31387	$+8.68124$	$+0.12287$	$+0.76738$	-9.50379	$-19.24=0$	2	$+0.19$
41	$+0.29959$	$+9.78104$	$+9.46389$	$+9.72509$	-0.21245	$+9.93095$	$+27.89=0$	1	-7.85
42	$+0.53986$	$+0.01870$	$+9.72591$	-9.07555	$+9.41330$	$+0.13226$	$+62.75=0$	1	-2.32
43	$+0.60823$	$+0.08565$	$+9.80414$	-9.63448	$+0.13194$	$+0.17551$	$+79.22=0$	1	$+2.39$
44	$+0.66629$	$+0.14145$	$+9.87506$	-9.90472	$+0.46135$	$+0.19479$	$+88.68=0$	2	$+3.21$
45	$+0.58906$	$+0.01662$	$+9.69548$	-8.77085	-9.89597	-0.17085	$+64.54=0$	2	-4.25
46	$+0.58053$	$+0.00604$	$+9.65801$	$+7.69897$	$+0.06296$	-0.18977	$+61.89=0$	5	-4.45
47	$+0.55443$	$+9.97359$	$+9.60746$	$+8.86923$	$+0.37310$	-0.17435	$+58.11=0$	3	-3.11
48	$+1.11212$	$+0.48615$	-0.07591	$+8.92942$	-9.56348	-0.36642	$+206.82=0$	5	-3.01
49	$+1.17566$	$+0.54888$	-0.14613	-9.26951	$+0.60531$	-0.38881	$+239.50=0$	5	-3.90
50	$+1.21383$	$+0.58614$	-0.19089	-9.60638	$+0.07882$	-0.37401	$+264.49=0$	2	-1.41
51	$+1.23252$	$+0.60325$	-0.21880	-9.80754	$+0.40037$	-0.27784	$+274.37=0$	2	-4.62
52	$+1.22025$	$+0.58973$	-0.21165	-9.85612	$+0.52595$	-0.15229	$+267.18=0$	3	-1.84
53	$+1.20238$	$+0.57089$	-0.19645	-9.86451	$+0.59073$	-0.02449	$+254.47=0$	2	-3.25
54	$+1.17507$	$+0.54245$	-0.17114	-9.86747	$+0.67265$	-9.78319	$+242.74=0$	2	$+1.46$

Normal Equations.

+ 102049·921δN	− 23867·784δε	+ 5939·191δπ	+ 17008·243δΩ	+ 900·111δi	+ 26541·270δφ	− 256336·626 = 0
− 23867·784	+ 7213·550	− 1803·145	− 4680·678	+ 237·062	− 7161·586	+ 125136·019 = 0
+ 5939·191	− 1803·145	+ 576 320	+ 1097·233	− 194·644	+ 2298 542	− 31261·762 = 0
+ 17008·243	− 4680·678	+ 1097·233	+ 3475·270	+ 440·675	+ 4103·138	− 72149·789 = 0
+ 900·111	+ 237·062	− 194·644	+ 440·675	+ 1492·595	− 1421·714	+ 13545·549 = 0
+ 26541·270	− 7161·586	+ 2298·542	+ 4103·138	− 1421·714	+ 11956·020	− 102418·880 = 0

Whence

δN = − 7″·160	with weight	13392·2	
δε = − 38·289	„	„	315·3
δπ = − 0·522	„	„	66·5
δΩ = + 4″·010	with weight	156·2	
δi = + 0·351	„	„	627·
δφ = + 0·293	„	„	1669·1

The corrections to the assumed elements of *Juno* are therefore:—

$dn = +\ 0''\cdot00716$	$d\Omega = -4''\cdot01$
$d\varepsilon = +38\cdot29 + dn \times t$	$di = -0\cdot35$
$d\pi = +\ 0\cdot52$	$d\varphi = -0\ 29$

where t is the number of days from 1876 March 19$^{d}\cdot0$ G.M.T.

From these corrections to the assumed elements of *Juno*, places near the time of each opposition have been computed for the transit at Greenwich, and from the observed errors of the uncorrected ephemerides (as given in the *Nautical Almanacs* of the different years) the following errors of R.A. and N.P.D. (given in the columns headed "Corrected") have been formed, by comparing the observed and computed places, thus showing the errors of places computed from the corrected elements :—

Date.	$\delta\alpha$. Uncorrected.	$\delta\alpha$. Corrected.	$\delta\alpha \cos\delta$. Corrected.	δ N.P.D. Uncorrected.	δ N.P.D. Corrected.
	s	s	″	″	″
1864 June 8	− 0·53	−0·04	− 0·6	− 1·2	−0·2
1865 Sept. 13	− 1·88	−0·12	− 1·8	+ 2·4	−0·5
1867 Mar. 1	− 1·58	−0·21	− 3·2	− 1·8	+0·1
1868 May 18	− 1·03	−0·02	− 0·3	− 0·9	+0·9
1869 July 30	− 2·06	−0·04	− 0·6	+ 2·0	+0·4
1871 Jan. 27	− 4·01	−0·18	− 2·7	+ 0·9	+1·7
1872 April 21	− 1·75	−0·03	− 0·5	− 4·4	−0·3
1873 June 21	− 2·26	−0·05	− 0·8	− 1·1	−0·4
1874 Nov. 13	− 8·33	−0·70	−10·5	+15·1	−0·3
1876 Mar. 31	− 2·97	−0·18	− 2·7	− 6·6	+0·8
1877 June 6	− 2·60	−0·11	− 1·7	− 3·6	−0·2
1878 Aug. 24	− 6·25	−0·13	− 2·0	+14·5	+0·8
1880 Feb. 21	− 4·98	+0·17	+ 2·6	− 8·9	+0·4
1881 May 10	− 2·91	+0·09	+ 1·4	− 5·5	+1·1
1882 July 16	− 4·63	+0·13	+ 2·0	+ 3·9	−0·3
1884 Jan. 13	−10·15	+0·22	+ 3·3	+ 3·5	+0·9
1885 April 19	− 3·76	+0·25	+ 3·8	− 8·9	+2·0
1886 June 13	− 4·04	+0·27	+ 4·1	− 2·2	−0·2
1887 Oct. 12	−14·02	+0·15	+ 2·3	+36·5	−0·9

The assumed elements of *Juno*, from which the *Nautical Almanac* ephemerides during the years mentioned above have been computed, were deduced by Mr. Hind from observations made near the times of twelve oppositions from 1841–55. The discussion is published in the *Nautical Almanac* for 1859.

An inspection of the errors exhibited in the above table will show that the corrected elements give a very good representation of the observed places during the years 1864–1887, with the exception of those observed in the year 1874, which give a discordant result in R.A.* It will be remarked that the signs of the

* The observed place in this case depends on 5 accordant observations, and leads one to suspect an error—possibly in the application of the perturbations—in the ephemeris. I have made inquiries on this point, but am informed that no such error can be detected. I have not, therefore, felt justified in rejecting this normal place.

errors of R.A. are negative in the earlier years, and positive in the later years, as if the mean motion required still further correction. It must be pointed out, however, that the comparison between the observed and computed places has not been made with any attempt at great accuracy, but rather as a general check on the correctness of the work; and that, in particular, the perturbations have not been recomputed as, of course, they ought to have been had the intention been to deduce a *definitive* orbit of the planet. To that the present paper makes no pretension, the object being merely to deduce corrections to the assumed elements of the orbit so as to obtain elements sufficiently accurate for the purpose of computing ephemerides for some years to come. And, as far as can be seen, that object has been attained.

My thanks are due to the Council of the Royal Society for a grant to defray the expenses of the computations, the results of which are embodied in this paper.

Blackheath:
1890 *June* 4.

On Star-Correction Tables. By W. H. Finlay, M.A.

The computation of star-corrections forms no small part of the reductions for the formation of a catalogue, as every astronomer who has taken part in such work knows well; so that any attempt to shorten the time required for these computations, without sacrificing accuracy or simplicity, is worthy of consideration. I think that this may be satisfactorily done to a considerable extent by tables in the following way. We have for any star—

$$\alpha' - \alpha = f + g \sin(G+\alpha) \tan\delta + h \sin(H+\alpha) \sec\delta,$$
$$\delta' - \delta = \imath \cos\delta + g \cos(G+\alpha) + h \cos(H+\alpha) \sin\delta.$$

Now I put—

$$P = \tfrac{1}{15} \cdot g_0 \sin(G+\alpha) \tan\delta, \qquad P' = g_0 \cos(G+\alpha),$$
$$Q = \tfrac{1}{15} \cdot h_0 \sin(H+\alpha) \sec\delta, \qquad Q' = h_0 \cos(H+\alpha) \sin\delta,$$

and

$$I = \imath \cos\delta,$$

where g_0 and h_0 are arbitrarily chosen constants; and I tabulate the values of P, Q, Q′ for each degree of declination and for every two minutes of time. The value of P′, which is independent of the declination, is given in a separate table for every minute (or smaller interval if desired) of time.

The star-corrections for any given date are then—

$$\alpha' - \alpha = \tfrac{1}{15} \cdot f + \mathrm{P} \cdot \frac{g}{g_0} + \mathrm{Q} \cdot \frac{h}{h_0} = \tfrac{1}{15} \cdot f + \mathrm{P}\left(1 + \frac{g - g_0}{g_0}\right) + \mathrm{Q}\left(1 + \frac{h - h_0}{h_0}\right)$$

(in time)

$$= \tfrac{1}{15} \cdot f + \mathrm{P}(1 + x) + \mathrm{Q}(1 + y),$$

and, similarly,

$$\delta' - \delta = \mathrm{I} + \mathrm{P}'(1 + x) + \mathrm{Q}'(1 + y),$$

where

$$x = \frac{g - g_0}{g_0}, \text{ and } y = \frac{h - h_0}{h_0}.$$

I is also tabulated for each degree of declination for values of ι, proceeding by differences of $0''{\cdot}1$.

The values chosen for g_0 and h_0 are $20''{\cdot}053$ and $18''{\cdot}5$ respectively, so that x is generally negative and never greater numerically than unity, while y is always positive and never greater than $0{\cdot}11$. The terms $\mathrm{P}x$, $\mathrm{P}'x$, $\mathrm{Q}y$ and $\mathrm{Q}'y$ can therefore be easily taken from Crelle's tables.

P, Q, P′, Q′ are taken from the tables with the argument $\mathrm{G} + \alpha$ for P and P′, and $\mathrm{H} + \alpha$ for Q and Q′. The values for the nearest whole minute of time will generally be amply sufficient in accuracy, but closer interpolation (if desired) is very simple. The increments in the values of P, Q, and Q′ for 1° of declination are given down the columns, and their variations for every fraction of a degree can be written at once under their tabular values. The signs of the terms are taken from a small table with the arguments $\mathrm{G} + \alpha$ and $\mathrm{H} + \alpha$; the upper sign, where two occur together, being for northern, and the lower for southern, declinations.

As an example, I give a portion of the table for declination 40°, in which the small figures at the side of each column give the change of the function for 1° of declination:—

	G+α P	G+α P′	H+α Q	H+α Q′
0.—VI.	±	+	+	±
VI.—XII.	±	−	+	∓

	G+α P	G+α P′	H+α Q	H+α Q′
XII.—XVIII.	∓	−	−	∓
XVIII.—XXIV.	∓	+	+	±

40°

		G+α P s	H+α Q s	H+α Q′ ″			ι ″	I ″
	…	…	…	…	…	↑ XI. XXIII.	…	…
I. XIII.	0	0·290	0·417	11·49	0	↑ X. XXII.	3·6	2·76
↓	2	0·300	0·430	11·46 (24)	58		3·7	2·83 (−4)
	4	0·309 (11)	0·444 (7)	11·43	56		3·8	2 91
	6	0·319	0·457	11·40	54		3·9	2·99
	8	0·328	0·471	11·37	52		4·0	3·06
	10	0·337 (12)	0·484	11·34	50		4·1	3·14
	12	0·347	0·498	11·31 (23)	48		4·2	3·22
	14	0·356 (13)	0·511	11·28	46		4·3	3·29 (−5)
	16	0·365	0·524 (8)	11·24	44		…	…
	18	0·374	0·537	11 21	42		…	…
	20	0·384	0 551	11·17	40		7·8	5·97
	22	0·393 (14)	0·564	11·14	38		7·9	6·05 (−9)
	24	0·402	0·577	11·10	36		8·0	6·13
	26	0·411	0·590	11·06	34		8·1	6·20
	28	0·420 (15)	0 603 (9)	11·03	32		8·2	6·28
	…	…	…	…	…	↑ X. XXII.		
	…	…	…	…	…			

		G+α P s	H+α Q s	H+α Q′ ″			ι ″	I ″
	…	…	…	…	…	↑ VIII. XX.	…	…
IV. XVI.	0	0·971	1·394 (21)	5·95 (12)	0		3·6	2·76
↓	2	0·976 (35)	1·401	5·86	58		3·7	2·83 (−4)
	4	0·981	1·408	5·77	56		3·8	2 91
	6	0·986	1·415	5·67	54		3·9	2·99
	8	0·990	1·422	5·58	52		4·0	3·06
	10	0·995 (36)	1·428	5·49	50		4·1	3·14
	12	0 999	1·435 (22)	5·40 (11)	48		4·2	3·22
	…	…	…	…	…		4·3	3·29 (−5)
	…	…	…	…	…		…	…
	40	1·054	1 513	4·07 (8)	20		…	…
	42	1·057 (38)	1·518 (23)	3·97	18		7·8	5·97
	44	1·061	1·522	3·87	16		7·9	6·05 (−9)
	46	1·064	1·527	3·77	14		8·0	6·13
	48	1·067	1·531	3·67	12		8·1	6·20
	50	1·070	1·535	3·58	10		8·2	6·28
	…	…	…	…	…	↑ VII. XIX.		

Now with this table let us find the star-corrections for—

Ex. 1. β Bootis, 1889 May 20.

From the American Ephemeris we get for this date—

f (*in time*) $= +0^{s}\cdot114$, G $= 4^{h}\ 52^{m}\cdot3$, H $= 13^{h}\ 50^{m}\cdot5$, $\iota = -4''\cdot03$,

and—

x is found to be $= -\cdot869$, $y = +\cdot084$.

The computation is then as follows:—

1889 *May* 20.

$\alpha = 14^{h}\ 57^{m}\cdot8$	$\alpha = 14^{h}\ 57^{m}\cdot8$	$\delta = +40^{\circ}\ 50'$
G $= 4\ 52\cdot3$	H $= 13\ 50\cdot5$	
G $+\alpha = 19\ 50\cdot1$	H $+\alpha = 4\ 48\cdot3$	

R.A.			Dec.		
P_0	$0^{s}\cdot995$		P_0'	$9''\cdot26$ (*from separate table*)	
δP	30		$P'x$	$-8\cdot05$	
sum	1·025				
Px	$-\cdot891$				
		$(1) = -0^{s}\cdot134$			$(1) = +1''\cdot21$
Q_0	1·532		Q_0'	3·67	
δQ	19		$\delta Q'$	6	
Qy	+130		$Q'y$	+31	
		$(2) = +1\cdot681$			$(2) = +4\cdot04$
f		$(3) = +0\cdot114$	I	$3\cdot09-\cdot03$	$(3) = -3\cdot06$
sum		$+1\cdot661$	sum		$+2\cdot19$

Ex. 2. * 1889 March 12.

For this date—

$f = -0^{s}\cdot369$, G $= 9^{h}\ 5^{m}\cdot0$, H $= 18^{h}\ 31^{m}\cdot1$, $\iota = -8''\cdot08$,

and—

x is found to be $= -\cdot835$, $y = +\cdot015$.

$\alpha = 4^{h}\ 8^{m}\cdot1$	$\alpha = 4^{h}\ 8^{m}\cdot1$	$\delta = -40^{\circ}\ 24'$
G $= 9\ 5\cdot0$	H $= 18\ 31\cdot1$	
G $+\alpha = 13\ 13\cdot1$	H $+\alpha = 22\ 39\cdot2$	

R.A.			Dec.		
P_0	$0^{s}\cdot352$		P_0'	$19''\cdot04$ (*from separate table*)	
δP	5		$P'x$	$-15\cdot90$	
sum	·357				
Px	$-\cdot298$				
		$(1) = +0^{s}\cdot059$			$(1) = -3''\cdot14$
Q_0	0·557		Q_0'	11·16	
δQ	3		$\delta Q'$	9	
Qy	8		$Q'y$	17	
		$(2) = -0\cdot568$			$(2) = -11\cdot42$
f		$(3) = -0\cdot369$	I	$6\cdot19-\cdot04$	$(3) = -6\cdot15$
sum		$-0\ 878$	sum		$-20\cdot71$

The signs are not to be placed before the quantities as taken from the tables, but are inserted before the final values in the last column—viz. after (1), (2), and (3); the sign of I is always the same as that of ι. The quantities δP, δQ, $\delta Q'$ are the increments of P, Q, Q′ for the minutes of declination (in our case 50 for Ex. 1, 24 for Ex. 2), and are always additive if P, Q, and Q′ be taken from the table for the nearest degree *numerically* less than the star's declination, while the correction to I is subtractive.

A computation of one pair of these star-corrections with four-figure logarithms would require 160 figures and symbols; the ordinary method ($Aa+Bb+$&c.) requires 217 figures, without taking into account the labour of forming a, b, &c.; while the result above has been obtained with 110 figures. This means a very considerable saving of time when we consider that some thousands of these computations have to be made every year. It might be more convenient, in the case of only a few star-corrections being wanted, to have the values of P′ given in the tables with the others; but if many stars are being reduced, the first step would be to run in the values of P′ for all of them as soon as the values of $G+\alpha$ are formed.

At declination 60° the change of P for 1° is $0^s{\cdot}096$, which is perhaps too large for δP to be readily written down; the change of Q is somewhat less; so that it may be advisable to reduce the intervals of declination for which the tables, after 50°, are computed to half degrees. As a matter of fact, the star-corrections are practically only required to the nearest hundredth in R.A. and the nearest tenth in declination; and I do not anticipate any difficulty in using the tables as far as 70°. They will certainly save much time when the declination is small.

The value chosen for g_0 is the constant n of the precession for the year 1900, so that for many years the precessions in right ascension and declination for any star can be taken at once from the proper declination table with the argument R.A. —viz. $3^s{\cdot}073+P$ and P′, due regard being paid to the signs of P and P′. But, as far as concerns the computation of star-corrections, the tables hold good for all time, any changes in the astronomical constants being taken account of in the data of the Ephemeris and in the values of x and y.

For the most effective use of the tables the values of f, G, H should be given in time and ι in seconds of arc (as is done in the American Ephemeris), and the values of x and y for each day in the year. These two last might, however, be readily taken from a table proceeding by increments of $0''{\cdot}1$ of g and h.

I may mention that approximate tables of this kind were prepared by means of Crelle's tables, and used for a check-computation of the star-corrections for the *Victoria* and *Sappho* stars observed here last year, and were found to be satisfactory; but in order to secure accuracy in the last decimal place of the

tables a more exact calculation is necessary. I have made considerable progress in the calculation of accurate tables, and hope soon to have them completed.

Cape of Good Hope;
1890 *May* 5.

On the Computation of the Equation of the Centre in Elliptical Orbits of Moderate Eccentricities. By A. Marth.

Though the indirect solution of Kepler's problem is a sufficiently easy proceeding when only a limited number of positions have to be computed, it takes rather too much time and becomes tiresome when it has to be repeated periodically hundreds of times. To provide for such cases some method is required for deducing the true anomalies directly from the mean anomalies with the desired degree of accuracy.

To find the true anomalies with the accuracy sufficient for approximate ephemerides of the minor planets, two sets of general tables have been computed: Mr. Godward's "Auxiliary Tables for Computing an Approximate Ephemeris of a Minor Planet" . . . printed 1866, and used in the *Nautical Almanac* office, and MM. Callandreau and Fabry's "Tables numériques destinées a faciliter le calcul des éphémerides" . . . published, 1885, in the second volume of the *Bulletin Astronomique.*

To get the true anomalies with an accuracy required for closer computations, the late E. Schubert gives in his later tables and papers on the elements of the minor planets a special "Table for the correction to be added to the auxiliary anomaly v'," this v' being found by the equation

$$\cot \tfrac{1}{2} v' = \frac{1-e}{1+e} \cot \tfrac{1}{2} \text{ mean anomaly.}$$

The present paper is intended to furnish the means for facilitating the calculation of certain tables, which will allow the differences between the true and mean anomalies to be easily deduced, whenever the intended work may make the construction of such tables desirable.

If e denotes the eccentricity, μ and v the mean and true anomaly, ω the angular value of the radius, the equation of the centre is given by the known series (all terms to e^{10} inclusive being retained) as follows:—

$$v-\mu = 2\omega\left(1-\frac{1}{8}e^2+\frac{5}{192}e^4+\frac{107}{9216}e^6+\frac{6217}{737280}e^8+\ldots\right).e\sin\mu.$$

$$+\frac{5}{4}\omega\left(1-\frac{11}{30}e^2+\frac{17}{240}e^4+\frac{43}{7200}e^6+\frac{677}{86400}e^8+\ldots\right).e^2\sin 2\mu.$$

$$+\frac{13}{12}\omega\left(1-\frac{129}{208}e^2+\frac{285}{1664}e^4-\frac{973}{66560}e^6+\ldots\right).e^3\sin 3\mu.$$

$$+\frac{103}{96}\omega\left(1-\frac{451}{515}e^2+\frac{4123}{12360}e^4-\frac{1619}{25956}e^6+\ldots\right).e^4\sin 4\mu.$$

$$+\frac{1097}{960}\omega\left(1-\frac{29785}{26328}e^2+\frac{824605}{1474368}e^4-\ldots\right).e^5\sin 5\mu.$$

$$+\frac{1223}{960}\omega\left(1-\frac{23739}{17122}e^2+\frac{116265}{136976}e^4-\ldots\right).e^6\sin 6\mu.$$

$$+\frac{47273}{32256}\omega\left(1-\frac{12412897}{7563680}e^2+\ldots\right).e^7\sin 7\mu.$$

$$+\frac{556403}{322560}\omega\left(1-\frac{9490966}{5007627}e^2+\ldots\right).e^8\sin 8\mu.$$

$$+\frac{10661993}{5160960}\omega.e^9\sin 9\mu+\frac{7281587}{2903040}\omega.e^{10}\sin 10\mu+\ldots$$

$$=k_1.e\sin\mu+k_2.e^2\sin 2\mu+k_3.e^3\sin 3\mu+k_4.e^4\sin 4\mu+\ldots$$

$$=k_1.e\sin\mu.(1+\kappa.e\cos\mu)+k_2.e^3\sin 3\mu+k_4.e^4\sin 4\mu+\ldots,$$

where $\kappa=\frac{5}{4}\left(1-\frac{29}{120}e^2+\frac{7}{480}e^4+\frac{571}{230400}e^6+\frac{23657}{11059200}e^8\right)$.

If a table containing the values of

$$k_2.e^3\sin 3\mu+k_4.e^4\sin 4\mu+\ldots,$$

were prepared, the equation of the centre would be found by computing $k_1.e\sin\mu\,(1+\kappa.e\cos\mu)$ for each individual mean anomaly, and adding the corresponding tabular value representing the remainder of the series. However, as this remainder would be obviously smaller in a series in which the first term, instead of being multiplied by a factor $1+\kappa e\cos\mu$, had a divisor of the form $1-ce\cos\mu$, it will clearly be of advantage to convert the series for the equation of the centre into another, beginning with a term of the form

$$\frac{Ce\sin\mu}{1-ce\cos\mu},$$

in which the values of C and c are so determined that the first two terms of the old series are incorporated in the new expression. These values are accordingly to be found.

$$\frac{\sin\mu}{1-ce\cos\mu}=\sin\mu\,(1+ce\cos\mu+c^2e^2\cos^2\mu+c^3e^3\cos^3\mu+c^4e^4\cos^4\mu+\ldots)$$

But

$$4 \sin\mu\cos^2\mu = \sin\mu + \sin 3\mu.$$
$$16 \sin\mu\cos^4\mu = 2\sin\mu + 3\sin 3\mu + \sin 5\mu.$$
$$64 \sin\mu\cos^6\mu = 5\sin\mu + 9\sin 3\mu + 5\sin 5\mu + \sin 7\mu.$$
$$256 \sin\mu\cos^8\mu = 14\sin\mu + 28\sin 3\mu + 20\sin 5\mu + 7\sin 7\mu + \sin 9\mu.$$
$$8 \sin\mu\cos^3\mu = 2\sin 2\mu + \sin 4\mu.$$
$$32 \sin\mu\cos^5\mu = 5\sin 2\mu + 4\sin 4\mu + \sin 6\mu.$$
$$128 \sin\mu\cos^7\mu = 14\sin 2\mu + 14\sin 4\mu + 6\sin 6\mu + \sin 8\mu.$$
$$512 \sin\mu\cos^9\mu = 42\sin 2\mu + 48\sin 4\mu + 27\sin 6\mu + 8\sin 8\mu + \sin 10\mu.$$

Hence, putting for greater simplicity $ce = 2\eta$,

$$\begin{aligned}\frac{\sin\mu}{1 - ce\cos\mu} &= (1 + \eta^2 + 2\eta^4 + 5\eta^6 + 14\eta^8 + \ldots)\,.\,\sin\mu.\\ &+ (1 + 2\eta^2 + 5\eta^4 + 14\eta^6 + 42\eta^8 + \ldots)\,.\,\eta\sin 2\mu.\\ &+ (1 + 3\eta^2 + 9\eta^4 + 28\eta^6 + \ldots)\,.\,\eta^2\sin 3\mu.\\ &+ (1 + 4\eta^2 + 14\eta^4 + 48\eta^6 + \ldots)\,.\,\eta^3\sin 4\mu.\\ &+ (1 + 5\eta^2 + 20\eta^4 + \ldots)\,.\,\eta^4\sin 5\mu.\\ &+ (1 + 6\eta^2 + 27\eta^4 + \ldots)\,.\,\eta^5\sin 6\mu.\\ &+ (1 + 7\eta^2 + \ldots)\,.\,\eta^6\sin 7\mu.\\ &+ (1 + 8\eta^2 + \ldots)\,.\,\eta^7\sin 8\mu + \eta^8\sin 9\mu + \eta^9\sin 10\mu + \ldots\end{aligned}$$

and therefore

writing θ for $\eta\,(1 + \eta^2 + 2\eta^4 + 5\eta^6 + 14\eta^8 + \ldots)$,

$$\frac{Ce\sin\mu}{1 - ce\cos\mu} = \frac{2C}{c}(\theta\sin\mu + \theta^2\sin 2\mu + \theta^3\sin 3\mu + \theta^4\sin 4\mu + \ldots).$$

The value of c must accordingly be so chosen that

$$\theta = \frac{k_2}{k_1}\,.\,e = \frac{5}{8}e\left(1 - \frac{29}{120}e^2 + \frac{7}{480}e^4 + \frac{571}{230400}e^6 + \frac{23657}{11059200}e^8\right),$$

and C so that

$$\frac{2C}{c}\,.\,\theta = k_1.$$

The determination of the two values which satisfy these conditions gives—

$$C = 2\omega\left(1 - \frac{33}{64}e^2 + \frac{1705}{4096}e^4 - \frac{666617}{2359296}e^6 + \frac{18106717}{83886080}e^8 \ldots\right).$$

$$c = \frac{5}{4}\left(1 - \frac{607}{960}e^2 + \frac{27671}{61440}e^4 - \frac{19289249}{58982400}e^6 + \frac{2688060283}{11324620800}e^8 \ldots\right).$$

Hence follows—

$$\frac{Ce\sin\mu}{1-ce\cos\mu} = 2\omega\left(1-\frac{1}{8}e^2+\frac{5}{192}e^4+\frac{107}{9216}e^6+\frac{6217}{737280}e^8+\ldots\right).e\sin\mu.$$
$$+\frac{5}{4}\omega\left(1-\frac{11}{30}e^2+\frac{17}{240}e^4+\frac{43}{7200}e^6+\frac{677}{86400}e^8+\ldots\right).e^2\sin 2\mu.$$
$$+\frac{25}{32}\omega\left(1-\frac{73}{120}e^2+\frac{1253}{7200}e^4-\frac{3229}{230400}e^6\ldots\right).e^3\sin 3\mu.$$
$$+\frac{125}{256}\omega\left(1-\frac{17}{20}e^2+\frac{537}{1600}e^4-\frac{107939}{1728000}e^6\ldots\right).e^4\sin 4\mu.$$
$$+\frac{625}{2048}\omega\left(1-\frac{131}{120}e^2+\frac{889}{1600}e^4\ldots\right).e^5\sin 5\mu.$$
$$+\frac{3125}{16384}\omega\left(1-\frac{4}{3}e^2+\frac{1201}{1440}e^4\ldots\right).e^6\sin 6\mu.$$
$$+\frac{15625}{131072}\omega\left(1-\frac{63}{40}e^2\ldots\right).e^7\sin 7\mu.$$
$$+\frac{78125}{1048576}\omega\left(1-\frac{109}{60}e^2\ldots\right).e^8\sin 8\mu.$$
$$+\frac{390625}{8388608}\omega.e^9\sin 9\mu+\frac{1953125}{67108864}\omega.e^{10}\sin 10\mu+\ldots$$

the substitution of which in the old series for the equation of the centre furnishes the new series

$$v-\mu=\frac{Ce\sin\mu}{1-ce\cos\mu}+\{c_3e^3\sin 3\mu+c_4e^4\sin 4\mu+\ldots c_{10}e^{10}\sin 10\mu\},$$

in which the coefficients $c_3, c_4, c_5 \ldots$ have the following values

$$c_3=\frac{29}{96}\omega\left(1-\frac{151}{232}e^2+\frac{457}{2784}e^4-\frac{7207}{445440}e^6\ldots\right).$$
$$c_4=\frac{449}{768}\omega\left(1-\frac{8057}{8980}e^2+\frac{143047}{431040}e^4-\frac{300761}{4827648}e^6\ldots\right).$$
$$c_5=\frac{25729}{30720}\omega\left(1-\frac{707495}{617496}e^2+\frac{19386485}{34579776}e^4\ldots\right).$$
$$c_6=\frac{266213}{245760}\omega\left(1-\frac{2601092}{1863491}e^2+\frac{152311165}{178895136}e^4\ldots\right).$$
$$c_7=\frac{11117513}{8257536}\omega\left(1-\frac{732409783}{444700520}e^2\ldots\right).$$
$$c_8=\frac{545147297}{330301440}\omega\left(1-\frac{37265543611}{19625302692}e^2\ldots\right).$$
$$c_9=\frac{5335893541}{2642411520}\omega.$$
$$c_{10}=\frac{471668976257}{190253629440}\omega.$$

A comparison of these values with the values of the corresponding coefficients of the old series shows the advantage of the new series in the diminution of the coefficients.

The logarithms of the coefficients are, if m is the modulus,

$$\log C = \log 2\omega - \frac{33}{64} m \cdot e^2 + \frac{2321}{8192} m \cdot e^4 - \frac{268043}{2359296} m \cdot e^6 + \frac{77029999}{1006632960} m \cdot e^8.$$

$$\log c = \log \frac{5}{4} - \frac{607}{960} m \cdot e^2 + \frac{461681}{1843200} m \cdot e^4 - \frac{335831383}{2654208000} m \cdot e^6 + \frac{235312199009}{3397386240000} m \cdot e^8$$

$$\log c_3 = \log \left(\frac{29}{96}\omega\right) - \frac{151}{232} m \cdot e^2 - \frac{15391}{322944} m \cdot e^4 - \frac{466537}{374615040} m \cdot e^6.$$

$$\log c_4 = \log \left(\frac{449}{768}\omega\right) - \frac{8057}{8980} m \cdot e^2 - \frac{68350979}{967684800} m \cdot e^4 - \frac{214816003651}{40552444352000} m \cdot e^6.$$

$$\log c_5 = \log \left(\frac{25729}{30720}\omega\right) - \frac{707495}{617496} m \cdot e^2 - [8{\cdot}98109] m \cdot e^4.$$

$$\log c_6 = \log \left(\frac{266213}{245700}\omega\right) - \frac{2601092}{1863491} m \cdot e^2 - [9{\cdot}08903] m \cdot e^4.$$

or numerically, if ω is expressed in degrees,

$$\log C = 2{\cdot}059152628 - [9{\cdot}3501183] e^2 + [9{\cdot}0900695] e^4 - [8{\cdot}69321] e^6 + [8{\cdot}52157] e^8.$$

$$\log c = 0{\cdot}096910013 - [9{\cdot}4387018] e^2 + [9{\cdot}0365539] e^4 - [8{\cdot}73997] e^6 + [8{\cdot}47828] e^8.$$

$$\log c_3 = 1{\cdot}2382494 - [9{\cdot}4512733] e^2 - [8{\cdot}31592] e^4 - [6\ 73309] e^6.$$

$$\log c_4 = 1{\cdot}5250078 - [9{\cdot}5906814] e^2 - [8{\cdot}48680] e^4 - [7{\cdot}36183] e^6.$$

$$\log c_5 = 1{\cdot}6811243 - [9{\cdot}6968735] e^2 - [8{\cdot}61887] e^4.$$

$$\log c_6 = 1{\cdot}7928407 - [9{\cdot}7826127] e^2 - [8{\cdot}72682] e^4.$$

$$\log c_7 = 1{\cdot}88728 - [9{\cdot}8545] e^2.$$

$$\log c_8 = 1{\cdot}97573 - [9{\cdot}9163] e^2.$$

$$\log c_9 = 2{\cdot}06333.$$

$$\log c_{10} = 2{\cdot}15243.$$

Instead of the coefficients of e^2 . . . their logarithms are given enclosed in [ ] brackets, 10 being added to the characteristics.

If the values of C, c_3, c_4 . . . are required to be expressed in seconds, log 3600 must be added to the first terms, which become thus—

$\log C = 5{\cdot}615455129 - \ldots$ $\log c_5 = 5{\cdot}2374268 - \ldots$

$\log c_3 = 4{\cdot}7945519 - \ldots$ $\log c_6 = 5{\cdot}3491432 - \ldots$

$\log c_4 = 5{\cdot}0813103 - \ldots$ $\log c_7 = 5{\cdot}44358.$

I have given all terms to e^{10} inclusive, so that, if needed, they may be taken into account. The following table contains, for various values of log e, the values of the coefficients $c_7 e^7$, $c_8 e^8$, $c_9 e^9$, $c_{10} e^{10}$ expressed in degrees and also in seconds.

Coefficients expressed

log e	In Degrees. c_7e^7	c_8e^8	c_9e^9	$c_{10}e^{10}$	In Seconds of Arc. c_7e^7	c_8e^8	c_9e^9	$c_{10}e^{10}$
9·00	0·000008	·000001	...	...	0·03	0·003	...	...
·02	010	001	...	...	·04	·005	...	...
·04	014	002	...	...	·05	·007	...	...
·06	020	003	...	...	·07	·010	...	...
·08	027	004	...	...	·10	·014	...	...
9·10	·000038	·000006	001	...	0·14	0·021	0·003	...
·12	052	008	001	...	·19	·030	·005	...
·14	071	012	002	...	·26	·043	·008	...
·16	098	017	003	...	·35	·062	·011	...
·18	135	025	005	...	·49	·090	·017	...
9·20	·000186	·000036	007	001	0·67	0·13	0·026	0·005
·22	256	052	011	002	0·92	·19	·040	·008
·24	351	074	017	004	1·26	·27	·060	·013
·26	483	107	025	006	1·74	·38	·091	·020
·28	663	153	038	009	2·39	·55	·14	·03
9·30	·000909	·000220	058	014	3·27	0·79	0·21	0·05
·32	·001248	316	088	023	4·49	1·14	0·32	0·08
·34	·001710	453	133	036	6·16	1·63	0·48	0·13
·36	·002343	649	201	057	8·43	2·34	0·72	0·20
·38	·003207	930	304	090	11·55	3·35	1·10	0·32
9·40	0·004387	·001330	461	142	15·79	4·79	1·66	0·51

In case the desired accuracy allows the neglect of $c_7e^7 \sin 7\mu$, the computation of the tabular values of

$$\{c_3e^3 \sin 3\mu + c_4e^4 \sin 4\mu + c_5e^5 \sin 5\mu + c_6e^6 \sin 6\mu\}$$

for any value of log e and for each degree of μ is easily accomplished, if the required log sines of the multiples of μ are specially written out beforehand, so that the $30+22+18+15$ numerical values, which enter into combination, may be quickly computed.

If that is done for conveniently altered values of log e, the variation of the eccentricity may be easily taken into account. To save all trouble in computing the coefficients, their logarithms for suitable values of log e are supplied by the following tables.

In the computation of

$$\frac{Ce \sin \mu}{1 - ce \cos \mu}$$

the values of log C and log c must be properly interpolated to correspond to the varied values of log e. Want of space did not permit the differences of successive values to be given on the last pages.

log e	log C		log c		log c_2e^3		
8·00	2·059 1302	21	0·096 8826	27	5·238		
·02	1381	24	8799	29	5·298		
·04	1257	26	8770	32	5·358		
·06	1231	28	8738	35	5·418		
·08	1203	32	8703	38	5·478		
8·10	2·059 1171	34	0·096 8665	42	5·538		
·12	1137	37	8623	46	5·598		
·14	1100	42	8577	51	5·658		
·16	1058	45	8426	55	5·718		
·18	1013	49	8471	61	5·778		
8·20	2·059 0964	54	0·096 8410	66	5·8382		
·22	0910	60	8344	73	5·8982		
·24	0850	65	8271	80	5·9582		
·26	0785	72	8191	88	6·0182		
·28	0713	78	8103	96	6·0781	log c_4e^4	
8·30	2·059 0635	86	0·096 8007	105	6·1381	4·725	
·32	0549	94	7902	116	6·1981	4·805	
·34	0455	104	7786	127	6·2581	4·885	
·36	0351	113	7659	139	6·3181	4·965	
·38	0238	124	7520	152	6·3781	5·045	
8·40	2·059 0114	136	0·096 7368	167	6·4381	5·125	
·42	·058 9978	150	7101	183	6·4981	5·205	
·44	9828	164	7018	201	6·5580	5·285	
·46	9664	179	6817	220	6·6180	5·365	
·48	9485	197	6597	242	6·6780	5·445	
8·50	2·058 9288	216	0·096 6355	265	6·7380	5·525	
·52	9072	236	6090	290	6·7979	5·605	
·54	8836	260	5800	318	6·8579	5·685	
·56	8576	284	5482	349	6·9179	5·764	
·58	8292	312	5133	382	6·9778	5·844	log c_5e^5
8·60	2·058 7980	342	0·096 4751	420	7·0378	5·9244	4·680
·62	7638	374	4331	459	7·0978	6 0043	4·780
·64	7264	411	3872	504	7·1577	6·0843	4·880
·66	6853	450	3368	553	7·2177	6·1642	4·980
·68	6403	494	2815	606	7·2776	6·2441	5·080
8·70	2·058 5909	541	0·096 2209	664	7·3375	6·3240	5·180
·72	5368	593	1545	728	7·3975	6·4039	5·280
·74	4775	650	0817	798	7·4574	6·4838	5·380
·76	4125	713	·096 0019	875	7·5173	6·5637	5·479
·78	3412	781	·095 9144	959	7·5772	6·6436	5·579
8·80	2·0 8 2631		0·095 8185		7·6371	6·7235	5·679

log e	log C	log e	log c_3e^3	log c_4e^4	log c_5e^5	log c_6e^6
8·80	2·058 2631	0·095 8185	7·6371	6·7235	5·679	4·590
·81	·058 2213	·095 7672	7·6671	6·7634	5·729	4·650
·82	·058 1775	·095 7134	7·6970	6·8033	5 779	4·710
·83	·058 1316	·095 6571	7·7270	6·8432	5·829	4·770
·84	·058 0836	·095 5982	7·7569	6·8831	5·879	4·830
8·85	2·058 0334	0·095 5365	7·7868	6·9231	5·929	4·890
·86	·057 9808	·095 4719	7·8168	6·9630	5·9785	4·950
·87	·057 9257	·095 4042	7·8467	7·0029	6·0284	5·010
·88	·057 8681	·095 3334	7·8766	7·0428	6·0783	5·069
·89	·057 8078	·095 2593	7·9065	7·0827	6·1281	5·129
8·90	2·057 7446	0·095 1817	7·9365	7·1225	6·1780	5·189
·91	·057 6785	·095 1005	7·9664	7·1624	6·2278	5·249
·92	·057 6093	·095 0154	7·9963	7·2023	6·2777	5·309
·93	·057 5368	·094 9264	8·02620	7·2422	6·3275	5·368
·94	·057 4610	·094 8332	8·05610	7·2820	6·3773	5·428
8·95	2·057 3816	0·094 7356	8·08600	7·3219	6·4272	5·488
·96	·057 2985	·094 6335	8 11590	7·3618	6·4770	5·548
·97	·057 2116	·094 5266	8·14579	7·4016	6·5270	5·608
·98	·057 1205	·094 4146	8·17567	7·4415	6·5766	5·667
8·99	·057 0253	·094 2975	8·20555	7·4813	6·6264	5·727
9·00	2·056 9256	0·094 1748	8·23542	7·5211	6·6761	5·787
·01	·056 8212	·094 0465	8·26529	7·5609	6·7259	5·846
·02	·056 7120	·093 9121	8·29515	7·6007	6·7757	5·906
·03	·056 5977	·093 7714	8·32500	7·6405	6·8254	5·966
·04	·056 4781	·093 6242	8·35485	7·6803	6 8751	6·0255
9·05	2·056 3529	0·093 4701	8·38469	7·7201	6·9249	6·0852
·06	·056 2219	·093 3089	8·41452	7·7599	6·9746	6 1448
·07	·056 0848	·093 1401	8·44434	7·7996	7·0242	6·2045
·08	·055 9414	·092 9634	8·47416	7·8394	7·0739	6·2641
·09	·055 7913	·092 7785	8·50396	7·8791	7·1236	6·3237
9·10	2·055 6342	0·092 5850	8·53376	7·9188	7·1732	6·3832
·11	·055 4699	·092 3825	8·56355	7·9585	7·2229	6 4428
·12	·055 2980	·092 1706	8·59333	7·99823	7·2725	6·5023
·13	·055 1182	·091 9488	8·62310	8·03791	7·3221	6·5618
·14	·054 9300	·091 7167	8·65286	8 07757	7·3716	6 6213
9·15	2·054 7332	0 091 4739	8·68260	8 11722	7·4212	6·6807
·16	·054 5273	·091 2198	8·71233	8·15685	7·4707	6·7402
·17	·054 3119	·090 9539	8·74205	8·19647	7·5202	6·7996
·18	·054 0866	·090 6757	8·77176	8·23607	7·5697	6·8589
·19	·053 8510	·090 3846	8·80146	8·27564	7·6192	6·9183
9·20	2·053 6046	0·090 0801	8·83114	8·31520	7 6686	6·9776

log e	log C	log c	log c_3e^3	log c_4e^4	log c_5e^5	log c_6e^6
9·200	2·053 6046	0·090 0801	8·83114	8·31520	7·6686	6·9776
·205	·053 4771	·089 9226	8·84597	8·33497	7·6933	7·0072
·210	·053 3468	·089 7615	8·86080	8·35474	7·7180	7·0369
·215	·053 2136	·089 5968	8·87563	8·37450	7·7427	7·0665
·220	·053 0773	·089 4283	8·89045	8·39425	7·7674	7·0961
·225	·052 9380	·089 2559	8·90527	8·41400	7·7921	7·1257
·230	·052 7955	·089 0796	8·92008	8·43374	7·8167	7·1553
·235	·052 6498	·088 8993	8·93489	8·45348	7·8414	7·1849
·240	·052 5008	·088 7149	8·94969	8·47321	7·8661	7·2145
·245	·052 3485	·088 5263	8·96449	8·49294	7·8907	7·2441
9·250	2·052 1928	0·088 3334	8·97929	8·51266	7·9153	7·2736
·255	·052 0335	·088 1362	8·994081	8·53237	7·9400	7·3032
·260	·051 8707	·087 9344	9·008867	8·55207	7·9646	7·3327
·265	·051 7042	·087 7281	9·023648	8·57177	7·9892	7·3622
·270	·051 5340	·087 5171	9·038423	8·59146	8·01382	7·3918
·275	·051 3599	·087 3013	9·053194	8·61114	8·03842	7·4213
·280	·051 1820	·087 0807	9·067959	8·63082	8·06300	7·4508
·285	·051 0001	·086 8550	9·082719	8·65049	8·08758	7·4802
·290	·050 8141	·086 6243	9·097473	8·67015	8·11215	7·5097
·295	·050 6240	·086 3883	9·112221	8·68980	8·13670	7·5392
9·300	2·050 4297	0·086 1470	9·126963	8·70945	8·16125	7·5686
·305	·050 2310	·085 9002	9·141700	8·72908	8·18578	7·5981
·310	·050 0279	·085 6479	9·156430	8·74871	8·21031	7·6275
·315	·049 8203	·085 3899	9·171154	8·76833	8·23482	7·6569
·320	·049 6081	·085 1261	9·185871	8·78794	8·25932	7·6863
·325	·049 3911	·084 8563	9·200582	8·80754	8·28381	7·7157
·330	·049 1694	·084 5805	9·215286	8·82713	8·30829	7·7450
·335	·048 9428	·084 2985	9·229983	8·84671	8·33276	7·7744
·340	·048 7112	·084 0101	9·244673	8·86629	8·35721	7·8037
·345	·048 4744	·083 7153	9·259355	8·88585	8·38165	7·8330
9·350	2·048 2325	0·083 4139	9·274030	8·90540	8·40608	7·8623
·355	·047 9852	·083 1057	9·288698	8·92494	8·43050	7·8916
·360	·047 7325	·082 7906	9·303358	8·94447	8·45490	7·9209
·365	·047 4742	·082 4685	9·318010	8·96399	8·47928	7·9501
·370	·047 2103	·082 1392	9·332653	8·98360	8·50365	7·9794
·375	·046 9406	·081 8026	9·347288	9·002998	8·52801	8·00858
·380	·046 6650	·081 4584	9·361915	9·022483	8·55235	8·03778
·385	·046 3834	·081 1066	9·376633	9·041956	8·57668	8·06696
·390	·046 0957	·080 7470	9·391142	9·060417	8·60099	8·09612
·395	·045 8017	·080 3794	9·405742	9·080865	8·62528	8·12526
9·400	2·045 5014	0·080 0036	9·420332	9·100299	8·64956	8·15438

A simple Solution of Kepler's Problem. By A. Marth.

What is Kepler's problem? Is it the problem to find the eccentric from the mean anomaly *by means* of Kepler's equation? Or is it the problem to find somehow the eccentric anomaly which will *satisfy* that equation?

The simple method, which I now offer, for computing the eccentric anomaly ϵ from the mean anomaly μ, consists in substituting for Kepler's equation $\epsilon - \mu = e \sin \epsilon$ an equivalent equation, the solution of which is free from the difficulties with which that of Kepler's equation is beset, and which is derived from the latter by an extremely simple process.

The multiplication of

$$\epsilon - \mu = e \sin \epsilon \text{ by } \frac{\sin(\epsilon - \mu)}{\epsilon - \mu}$$

gives

$$\sin \epsilon \cos \mu - \cos \epsilon \sin \mu = e \sin \epsilon \cdot \frac{\sin(\epsilon - \mu)}{\epsilon - \mu};$$

Hence

$$\tan \epsilon = \frac{\sin \mu}{\cos \mu - e \cdot \dfrac{\sin(\epsilon - \mu)}{\epsilon - \mu}}$$

or

$$\tan \epsilon = \frac{\sin \mu}{\cos \mu - \dfrac{e}{\nu}}$$

if

$$\frac{1}{\nu} = \frac{\sin(e \sin \epsilon)}{e \sin \epsilon}$$

$$= 1 - \frac{e^2 \sin^2 \epsilon}{6} + \frac{e^4 \sin^4 \epsilon}{120} - \frac{e^6 \sin^6 \epsilon}{5040} + \frac{e^8 \sin^8 \epsilon}{362880} - \frac{e^{10} \sin^{10} \epsilon}{39916800} + \ldots$$

The equation $\mu = \epsilon - e \sin \epsilon$ retains its value for computing μ from ϵ, and may thus serve as a control upon the determination of $\tan \epsilon$. Except for the purpose of such control, the angle ϵ itself is not required.

The system of formulæ for deducing the true anomaly v and the radius vector ρ from the mean anomaly μ becomes

$$\left.\begin{aligned} \tan \epsilon &= \frac{\sin \mu}{\cos \mu - \dfrac{e}{\nu}} \\ \rho \sin v &= \sin \epsilon \cdot \sqrt{1 - e^2} \\ \rho \cos v &= \cos \epsilon - e. \end{aligned}\right\}$$

The values of $\log \nu$ corresponding to $\log (e \sin \epsilon)$ will be given in a table in the next number of the *Monthly Notices*.

On the Verification of the Constants employed in the Uranometria Nova Oxoniensis. By the Rev. Professor Pritchard, D.D., F.R.S.

In the *Monthly Notices* of March last Dr. Spitta throws some doubt as to the method employed in determining the coefficient of absorption of the Wedge Photometer used for the purposes of the Uranometria Oxoniensis. If I understand his criticism correctly, he doubts whether a proper diaphragm was applied to the Nicol prism in the apparatus constructed for deriving the coefficient aforesaid. He found, it appears, that in his own case the proper use of a proper diaphragm removed some sources of error, and that he thereby secured results coincident with others obtained by a different photometric process which he regards as free from error.

With regard to the accuracy of the two Oxford wedges described in the *Memoirs R.A.S.*, vol. xlvii., and used for the purposes of the Uranometria, I had in effect but little anxiety. These wedges are of very different steepness, and the coefficient of absorption of each was independently obtained from nearly three thousand distinct measures;* for I was more anxious to secure accuracy than to save time usefully expended. Still, after Dr. Spitta's remark, I felt it my duty to re-examine one of the two wedges; and I duly attended to his remarks as to the necessity of using a proper diaphragm in order to obviate the internal and other reflections which he regards as the source of error in his own case. The result was that no correction could possibly be made in the coefficient of absorption of the wedge used for the Oxford Uranometria. The method employed for the re-examination of this wedge was an improvement on that used in 1882, and one whereby any accumulation of small errors was avoided; which errors, if they existed, would arise from the smallness of the interval of the wedge, examined step by step by the old process of 1882. In fact it was to avoid these possible errors, which led to the employment of so many thousand applications of the instrument.

The results obtained by the two distinct methods of examination for the absorption (expressed in stellar magnitude) at each successive inch of the wedge is shown below.

* Moreover, I had observed that the Nicol Prism employed for obtaining this constant, was perfectly free from stilbite, or other silicates, and allowed no light whatever, perceptible to the eye, to pass through it when in the position of zero. I may here add, too, that a very scrupulous examination was instituted in regard to the absorptive power of the wedge in the case of red, orange, green, and blue lights. The absorptive power was practically the same for them all. See *Memoirs Roy. Ast. Soc.*, vol. xlvii. p. 395.

Inch.	1882. Mag.	1890. Mag.
1	1·93	1·93
2	3·81	3·89
3	5·69	5·62
4	7·52	7·53
5	9·30	9·41
6	11·07	10·95

Experiments were also made to ascertain the effects of diminishing the aperture of the diaphragm used between the eye and the Nicol. The result was that, as used at Oxford, any material alteration in the diameter of this aperture introduced errors which had not existed before. In order that no mistake should be made in the question of diaphragm, the assistance of Mr. Selby, the demonstrator in the Clarendon Physical Laboratory at Oxford, was invoked, and his independent conclusions coincided with those of Mr. Plummer and Mr. Jenkins.

I take it, therefore, that the coefficients of absorption of the two Oxford wedges are, so far, established as correct. But this is by no means the whole of the case.

Dr. Spitta's objections in reality do not apply solely, or even particularly, to the Wedge Photometer, but to every species of photometrical measure hitherto made by means of the properties of polarised light. They implicitly apply to instruments used by Zöllner, Lindemann, Ceraski, Pickering, and other physicists; but inasmuch as Dr. Spitta finds that on the use of proper precautions the method of polarisation does give the same results as those arising from what he regards as superior methods, I need say no more on the generic question of the instrument or the method used for the evaluation of the coefficient of absorption of a wedge.

I may add also that, in order to extend my own work to stars too faint for the two wedges hitherto used in the Oxford Observatory, I had requested Mr. Hilger to undertake the formidable task of constructing as thin a wedge as he, with all his skill, could effect. In due time I received this excellent instrument, and I found it now applicable to the extinction of stars of mag. $11\frac{1}{2}$, as used with the Oxford $12\frac{1}{4}$-inch object-glass. I proceeded to evaluate its coefficient of absorption by the new and improved method adopted by Professor Pickering (*Investigations on Light and Heat*, No. vi. p. 323). The result was that an interval of one inch extinguished 0·97 mag. I had previously requested Dr. Spitta kindly to evaluate the same wedge constant with his best care and his best methods. His result was that 1·12 mag. was extinguished at each successive inch of the wedge. Armed with these constants, this thin wedge was applied to the heavens, and Mr. Plummer carefully observed thirty stars of the *Pleiades* from magnitude 7 to 11. The resulting magnitudes

were compared with those assigned by Professor Pickering (*Annals Harvard Observatory*, vol. xviii. pt. vii.), and it was found that the average difference of magnitude (Harvard—Oxford) was only 0·07 mag. But, on the other hand, if I used the instrumental determination furnished by Dr. Spitta, the difference between the Harvard and Oxford magnitude would amount to as much as 0·54 mag. Such a difference is wholly unknown in any comparison of my works with those of the most eminent observers.

Again, a similar process was adopted with regard to twenty stars observed by Lindemann.* The result here was that with the Oxford value of absorption the average difference (Lindemann —Oxford) amounted to only −0·05 mag. With Dr. Spitta's constant this difference is increased from 0·05 to 0·50 mag. The photometer used by Professor Pickering was his own meridian photometer, aided by photography. Dr. Lindemann used the ordinary Zöllner photometer.

I think that here also I may properly appeal to the results which Dr. Lindemann and Professor Pickering have themselves deduced and recorded from the Oxford photometry. The former says: "From thirty-three stars of the *Pleiades* the mean deviation of a single determination of relative brilliancy equals (Pr—Lind) +0·05 mag." † The latter, from the more extensive comparison with the Uranometria Oxoniensis, says: "The mean result from the 2647 stars compared is, that the average magnitude of a star in the U. O. exceeds the corresponding magnitude in the H. P. by 0·049 mag." ‡

It appears, then, from the foregoing remarks, that both Lindemann and Pickering agree in their results with the Oxford determinations. Each of these employed a method and an instrument of his own, differing from each other and from the method employed at Oxford; and, moreover, this agreement among the three astronomers would be completely destroyed if Dr. Spitta's constant of absorption were adopted.

Nevertheless, I beg to express my sincere obligations to the latter gentleman for his inducing me to re-examine the whole question of wedge photometry, and I hope also he will permit me to acknowledge my admiration of the perseverance and ingenuity with which he has carried on his own investigation.

University Observatory, Oxford:
1890 *June* 12.

* Photometrische Bestimmung der Grössenclassen der Bonner Durchmusterung. St. Petersburg, 1889.

† *Op. cit.* p. 145. ‡ *Annals Harvard Observatory*, vol. xviii. p. 17.

Note on the Scaling of Dr. Spitta's Wedge by means of Photography.
By Captain W. de W. Abney, C.B., D.C.L., F.R.S.

Two or three meetings ago Dr. Spitta read a paper on the scaling of his wedge as used for astronomical purposes, and in the discussion which followed I said that wedges could also be scaled photographically. The scaling, I need scarcely say, referred to the determination of the absorption coefficient per unit of length. Dr. Spitta kindly placed his wedge at my disposal, and I must premise that until I had more than half completed my experiments I had no idea what the coefficient he had determined was, nor the unit of length for which he took it. On subsequently reading his paper in the *Monthly Notices*, I was still at a loss to know these points, as he had not expressly stated the latter, and had to ask him by letter what they were. The wedge he lent me is one cut out of what I suppose ought to be neutral tint, but which is really a species of bottle-green glass. Examining it in the spectrum it was easy to see that there was a much larger absorption coefficient in the red than in the green, and in the green than in the blue-green. Beyond that, it was unnecessary to examine the wedge in this manner. My first experiment was to determine its absorption for the rays which act on iron salts, the maximum of which lies in the blue-green rays. For this purpose I employed the ordinary platinum paper supplied by the Platinotype Company. A strip was placed in contact with the wedge and exposed to the direct light from an arc electric light, care being taken to prevent any light penetrating through the sides of it. The light was placed at such a distance from the wedge that practically the rays falling on it were parallel rays. Simultaneously, and with the proper precautions, a piece of platinum paper was exposed in a Spurge sensitometer, which I have described in a paper read before the Society last year. The papers were developed together and washed and dried, the paper having been previously marked off in $\frac{1}{4}$ inches, that all error due to shrinkage might be avoided. The sensitometer print was measured for blackness, and a curve constructed; the abscissæ used were the known intensities of light, and the ordinates the blackness, or perhaps I should say the amount of whiteness left. This was measured with my photometer in the way I have described here and in other papers. The wedge print was measured in the same way at every $\frac{1}{4}$ of an inch, and the intensities of light acting at every $\frac{1}{4}$ inch derived from the curve, the amount of whiteness indicating the intensity which had acted.

The measurements gave me the coefficient of absorption. This gave a star magnitude of 1·38 per inch.

Now evidently the value thus obtained would not be the same as the value to the eye, since it is the yellow part of the spectrum which has most effect on the latter and the

blue on the former. To obtain this value a solution of chromate of potash was placed in front of the light, and a Carbutt film of bromide of silver placed in contact with the wedge, a sensitometer containing the same sensitive surface being exposed simultaneously to the same light. This light gave the maximum photographic sensitiveness in the yellow-green, and accorded very well with that of the eye. The densities of the sensitometer negative and of the wedge negative were measured as before, and in this case the star magnitude per inch came out 1·20 to 1·21. Another experiment gave the value of 1·21 to 1·22, whilst a third gave the value 1 20 to 1 22. We may take it that the value does not much differ from 1·21. To determine the coefficient in red light, the wedge and sensitometer were exposed to light passing through what is known as stained red glass. It is a glass preferable to ruby glass, as this last allows a small percentage of blue rays to pass. Proceeding as before in measuring the negatives, in two experiments the values ·96 and ·93, or a mean of ·945, were obtained. It was only after the second set of experiments with the light passing through the chromate cell that I communicated with Dr. Spitta. He then informed me that his coefficients per inch gave him the following figures in star magnitudes :—

By method of reflection from mirrors	1·192
By rotating photometer	1·176
By polarising method with diaphragm	1·160
By „ „ without diaphragm	1·42

The only result of mine comparable with these is that obtained with light passing through the chromate solution, which gave me 1·21, a value a little higher than his value, 1·192.

I think this small discrepancy can be accounted for, if we take into consideration that he used gaslight—a light which contains a greater proportion of red and yellow than does sunlight; for, as has been stated, the red light is more absorbed than the yellow by this particular wedge. This being so, the maximum optical effect would be obtained in a part of the spectrum slightly nearer to the red than he would have obtained with sunlight.

The light I used was the electric light; and I have shown that the light from the positive pole of the electric light is very nearly the same as that of sunlight at noon about the middle of May. The results obtained with the electric light would therefore be very closely the same as that obtained with sunlight or with certain classes of stars.

The experiments I have carried out seem to point to this wedge having a different coefficient for different classes of stars. The difference should be most observable in red stars.

In conclusion, I may say that as well as films I tried glass

plates backed with a black material to avoid halation. I was not satisfied that halation would not throw out the accuracy of results somewhat. The experiments showed that my fears were groundless.

I think it satisfactory, at all events, for the photographic method of scaling, that Dr. Spitta's and my own coefficients come out so close as they do. The wedge seems to be very uniform in its coefficient of absorption throughout, as the results at the darker end were the same as those of the lighter.

Note on some Variable Stars near the Cluster 5 *M.*

By A. A. Common, F.R.S.

On comparing some photographs of the Cluster 5 Messier lately taken with the five-foot telescope, I was surprised to find that on one plate there were stars that were quite invisible on the other plates. Considerable variation in the size of the impressions of the stars on the plate is not at all unusual, but the appearance of new stars of considerable magnitude on one plate, without a trace of them on plates taken before and after, is worthy of notice. Four photographs of this Cluster have been taken this year—on April 22, May 9, May 15, and June 9, with 25, 45, 66, and 45 minutes' exposure respectively.

The plate taken on May 15, with the longest exposure, contains five stars not shown on the other plates; the position with regard to the centre of the Cluster is given below :—

No.	Mag.	*AR*	*δ*
1	9·5	20*p*	19′ N
2	10·0	10*f*	1·5 N
3	9·5	40*f*	12 S
4	10·0	56*f*	20 S
5	9·0	66*f*	10·5 N

The magnitude of the above stars is estimated from the 9·5 (Argelander) star about 18′ south of the Cluster.

The plate on which these stars appear has more stars than the other three plates, owing to the longer exposure, but stars of at least the 12th magnitude are shown on all of them; so that the presence of the new stars is not due to the longer exposure. On re-measuring the plates, to check the position of the five stars, I find that there is a great difference in the apparent magnitude of many of the stars near the Cluster, particularly with one star of about 10· magnitude, about 3ˢ *f* the centre of Cluster: on the 1st, 3rd, and 4th plates it is an unmistakable object; on the 2nd plate, taken May 9, it is there, but just the same magnitude as the other stars of the Cluster—possibly two magnitudes less than its magnitude on the other plates.

On the Variable Star U ("Nova") Orionis. By J. E. Gore.

From a reduction of all my observations of this variable star since the date of its discovery, 1885 December 13, I find the following epochs of maxima:—

1885 Dec. 13?	1889 Jan. 1
1886 „ 11	1890 „ 13
1887 „ 30	

Treating these by the method of least squares, I find the following elements:—

Maximum = 1887, Dec. 26·4 + 373^{d}·6 E.

As, however, it seems probable that the star had passed the maximum previous to the night of its discovery, I have thought it safer to neglect this maximum, and deduce elements from the following observation of maxima:—

1886 Dec. 11	Gore	1889 Jan. 1	Gore
„ 13	Sawyer	1890 Jan. 3	Yendell
1887 Dec. 30	Gore	„ 6	Sawyer
1888 Dec. 26	Sawyer	„ 13	Gore
„ 27·5	Yendell		

Treating these by the method of least squares, I find the following provisional elements:—

Maximum = 1887, Dec. 22·36 + 373·47 E.

The following is a comparison between the observed maxima and those computed from the above elements:—

Observed Maxima.	Computed Maxima.	O−C. days.
1885 Dec. 13	1885 Dec. 5·42	+7·59
1886 Dec. 11	1886 Dec. 13·89	−2·89
„ 13	„ 13·89	−0·89
1887 Dec. 30	1887 Dec. 22·36	+7·64
1888 Dec. 26	1888 Dec. 29·83	−3·83
„ 27·5	„ 29·83	−2·33
1889 Jan. 1	„ 29·83	+2·17
1890 Jan. 3	1890 Jan. 7·3	−4·3
„ 6	„ 7·3	−1·3
„ 13	„ 7·3	+5·7

These elements give the date of the next maximum 1891, January 15·77.

On the Proper Motions of Three Stars. By W. T. Lynn, B.A.

In vols. xxxi. and xxxiii. of the *Monthly Notices* I gave the proper motions, by the aid of a few then recent Greenwich observations, of three stars which had been found to have large motions. But as all three are contained in the new 10-year catalogue, it is now possible to deduce tolerably accurate values of the proper motions from Greenwich observations (all made with the same instrument and directly comparable) only. This I here proceed to do, and beg to offer the results to the Society, taking the stars in the order of right ascension.

1. The first is Lalande 21185 = W. B. (2) X. 1112. In my paper in the *Monthly Notices* for November 1872 (vol. xxxiii. p. 52), I gave the (corrected) values deducible from the places used in the two 7-year catalogues, the first of which was in both elements, the second in N.P.D. (in which element the proper motion is large) only. Adding to these the places given in the 9-year and the 10-year catalogues we obtain:—

Epoch.	R.A.	No. of Obs.	N.P.D.	No. of Obs.	Authority.
	h m s		° ′ ″		
1860	10 55 40·56	2	53 5 34·86	2	7-year Cat. (1860)
1864	...	0	53 7 9·24	2	7-year Cat. (1864)
1872	10 56 20·17	4	53 10 21·90	4	9-year Cat.
1880	10 56 46·49	5	53 13 34·61	5	10-year Cat.

The precession for 1860 is $+3^s·349$ (diminishing to $+3^s·344$ in 1880) in R.A. and $+19''·27$ (increasing to $+19''·30$) in N.P.D. From this it would seem that, as the amount of apparent change from 1860 to 1872 is $+39^s·61$ or $+3^s·301$ per annum in R.A., and $+4'·47''·04$ or $+23''·92$ per annum in N.P.D., the proper motion deducible from a comparison between the first 7-year and the 9-year catalogues is $-0^s·046$ in R.A. and $+4''·64$ in N.P.D. Comparing the places in the first 7-year and in the 10-year catalogues (separated by an interval of 20 years) we obtain similarly a proper motion of $-0^s·049$ in R.A. and $+4''·70$ in N.P.D.

2. The second of these stars is Lalande 21258 = W. B. (2) X. 1174, of which I gave the place (corrected) from the second 7-year catalogue in the *Monthly Notices* for December 1872 (vol. xxxiii. p. 102). Tabulating this together with the places in the 9-year and the 10-year catalogues, we obtain:—

Epoch.	R.A.	No. of Obs.	N.P.D.	No. of Obs.	Authority.
	h m s		° ′ ″		
1864	10 58 42·55	1	45 46 33·67	1	7-year Cat. (1864)
1872	10 59 6·74	3	45 48 58·33	3	9-year Cat.
1880	10 59 30·54	7	45 51 26·19	6	10-year Cat.

Apparent Mean Annual Variations.

Years.	Ann. Var. in R.A.	Ann. Var. in N.P.D.
	s	″
1864–1872	+3·024	+18·08
1872–1880	+2·975	+18·48

Now, as the annual precession is, for the mean epoch, $+3^{s}\cdot413$ in R.A. and $+19''\cdot35$ in N.P.D., we have, for the mean proper motions from the two comparisons, $-0^{s}\cdot413$ in R.A. and $-1''\cdot07$ in N.P.D.

3. The third and last of these stars is Oeltzen's Argelander 17415–6, of which I gave a provisional determination of the proper motion in the *Monthly Notices* for December 1870 (vol. xxxi. p. 42). It was not, as it happened, included in any great Greenwich catalogue until the 9-year; I can therefore now only compute its proper motion from Greenwich observations by comparing the places in that catalogue and in the recent 10-year. The former (where it is No. 1619) is R.A. $17^{h}\ 37^{m}\ 10^{s}\cdot302$, N.P.D. $21^{\circ}\ 32'\ 38''\cdot94$; number of observations, four in each element; epoch 1872. The latter (where it is No. 2786) is R.A. $17^{h}\ 37^{m}\ 7^{s}\cdot400$, N.P.D. $21^{\circ}\ 33'\ 4''\cdot15$; number of observations, four in each element; epoch 1880. This gives apparent annual variation $-0^{s}\cdot363$ in R.A. and $+3''\cdot15$ in N.P.D.; and as the precession for the mean epoch is $-0^{s}\cdot297$ in R.A. and $+2''\cdot00$ in N.P.D., we have, finally, for the deduced values of the proper motion in R.A. and N.P.D. respectively, $-0^{s}\cdot066$ and $+1''\cdot15$.

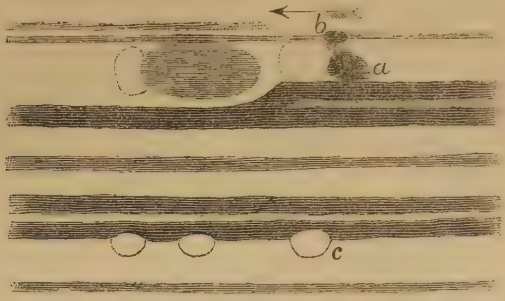
Blackheath:
1890 *April* 23.

On a coming Conjunction of a Remarkable Dark Spot on Jupiter with the Red Spot, and the relative Altitudes of these Objects. By A. Stanley Williams.

It is now twelve years since the great Red Spot made its appearance upon *Jupiter*, or at least since it first attracted general attention, and notwithstanding all the consideration that has been bestowed upon it during these twelve years, it still remains almost as much of an enigma as at its first appearance. Especially nothing is yet known for certain with regard to the altitude of the spot in the atmosphere of the planet, or with reference to neighbouring markings. A number of observations, it is true, have been made bearing upon this subject, but they are none of them decisive. Indeed, they differ curiously from each other. The object of this note is to call attention to an *unrivalled and unique opportunity* of obtaining conclusive evidence upon this important question, which is now on the point of occurring.

It is well known that most of the spots and markings of *Jupiter* situated to the south of the great equatorial system of

belts have a slow motion from east to west relative to the Red Spot. A remarkable dark spot is now visible on the south side of the South Equatorial Belt, which by virtue of this westerly motion or drift must in a short time overtake that marking, provided, of course, that the conditions continue to remain unchanged. The spot in question is a large, dark, very prominent and very striking feature, and is marked *a* in the accompanying figure, which represents the equatorial region of *Jupiter* as it



appeared about the end of May. It is evident that, if the spot continues to sweep onwards from east to west, in the direction of the arrow, it must eventually overtake and pass the Red Spot. In that case it must, if situate at a higher level, cover about half the breadth of the latter marking. If, on the other hand, it should lie at a lower level in the atmosphere of the planet, then the spot should disappear altogether from view. If at about the same level it would probably be diverted.

Since the year 1879 I have observed *Jupiter* and the Red Spot more or less continuously, and in all that time have seen no other spot of such a striking and conspicuous character and so magnificently suited to solve the vexed question of the altitude of the latter marking compared with neighbouring objects. I therefore venture to call the attention of observers to the importance of taking advantage of this *unrivalled opportunity*, which may never occur again. It should be mentioned that the dark spot *a* was first observed as long ago as the commencement of June in last year, and has consequently already been visible for more than twelve months. It still retains almost the same appearance which it possessed when first seen, and there is therefore every prospect of its continuing visible as a prominent object for a considerable time longer. The following are the approximate dates on which the dark spot *a* should be in conjunction with different parts of the Red Spot:—

With the following end	...	...	29 July 1890
With the centre	...	...	28 Aug. 1890
With the preceding end	...	...	27 Sept. 1890

From this it will be seen that the dark spot will take about two months to traverse the length of the Red Spot. It should be closely watched during all this time, but the chief interest obviously attaches to its first encounter with the following end of the latter. This will occur about the date of opposition, when *Jupiter* will be fairly well placed for observation, though his great south declination renders it very desirable that observations should be secured in more southern latitudes. The following are the observed times of transit of the dark spot over the central meridian of the planet's disc which have been obtained in the present year, with the intervals by which it followed Mr. Marth's zero meridian. They show clearly the westerly drift of the spot:—*

Date.	G.M.T. of Transit.	Following zero Meridian by.
	h m	h m
1890 Mar. 28	17 20	1 56·6
May 1	15 5·2	1 33·6
20	15 31·6	1 19·3
30	13 32	1 4·3

Besides the above-mentioned dark spot *a*, there is a little dark spot above it marked *b*. This is also worthy of notice, since in its passage westward it will just skirt the southern edge of the Red Spot, and may perhaps overlap it. Another noteworthy object is the white spot marked *c* in the figure, on the north edge of the North Equatorial Belt. This is remarkable chiefly for its long-continued visibility, since it was first seen as long ago as January 27, 1885, by Mr. Denning at Bristol,† and March 19, 1885, by Dr. Terby at Louvain.‡ It has been observed at every opposition since.

Percy Lodge, Burgess Hill:
1890 *June* 4.

* The centre of the red spot precedes this zero meridian at the present time by about 14 minutes.

† *Observatory*, 1885, p. 304.

‡ *Études sur l'aspect physique de la Planète Jupiter.* Deuxième Partie: pp. 35–37. The spot is designated ⧖ by Dr. Terby.

Observations of Comet a 1890 (*Brooks*) *made at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

The observations were made with the East or Sheepshanks Equatorial, aperture 6·7 inches, by taking transits over two cross-wires at right angles to each other, and each inclined 45° to the parallel of declination.

Comet a 1890 (*Brooks*).

Greenwich Mean Solar Time.	Observer.	✓−✶ R.A.	Corr. for Refraction.	✓−✶ N.P.D.	Corr. for Refraction.	No. of Comp.	Apparent R.A.	Log. factor of Parallax.	Apparent N.P.D.	Log. factor of Parallax.	Comp. Star.
d h m s		m s	s	′ ″	″		h m s		° ′ ″		
May 15 9 48 5	L.	−0 39·0	0·0	−10 57·3	−0·9	4	...	9·6751	...	0·7964	*a*
10 0 47	,,	−0 42·1	0·0	− 1 3·7	−0·1	8	...	9·6814	...	0·7823	*b*
10 11 50	,,	+1 37·0	0·0	+ 1 38·5	+0·1	3	20 35 42·0	9·6870	47 34 49·4	0·7700	*c*
10 21 58	,,	−0 11·0	0·0	− 7 2·2	−0·6	2	...	9·6911	...	0·7581	*d*
17 9 36 40	A. D.	+0 10·3	0·0	+ 6 18·7	+0·4	3	...	9·6901	...	0·7860	*e*
9 46 14	,,	−0 36·0	0·0	+ 6 27·6	+0·4	3	...	9·6956	...	0·7748	*f*
9 53 21	,,	−0 13·0	0·0	− 6 42·6	−0·4	2	...	9·6992	. .	0·7663	*g*
18 12 47 56	T.	+1 41·9	0·0	− 6 20·3	−0·1	6	20 26 53·1	9·6475	44 32 45·6	0·4415	*h*
12 50 58	,,	+1 23·2	0·0	− 5 19·3	−0·1	5	20 26 54·2	9·6440	44 32 52·5	0·4345	*i*
20 9 44 17	L.	+1 12·3	0·0	− 5 18·4	−0·2	2	20 20 44·4	9·7272	42 42 37·4	0·7330	*j*
9 44 17	,,	+0 4·3	0·0	+ 0 7·1	0·0	2	...	9·7272	...	0·7330	*k*
21 9 35 59	H.	−1 10·7	0·0	+ 0 51·0	0·0	3	20 17 11·5	9·7355	41 44 9·1	0·7276	*l*
9 46 8	,,	+0 25·2	0·0	− 0 3·4	0·0	4	20 17 10·0	9·7374	41 43 42·5	0·7119	*m*

Greenwich Mean Solar Time.		Observer.	♂−✶ R.A.	Corr. for Refraction.	♂−✶ N.P.D.	Corr. for Refraction.	No. of Comp.	Apparent R.A.	Log. factor of Parallax.	Apparent N.P.D.	Log. factor of Parallax.	Comp. Star.
	h m s		m s	s	′ ″	″		h m s		° ′ ″		
May 2	9 34 51	L.	+0 50·5	0·0	− 9 9·2	−0·2	2	20 13 24·1	9·7461	40 45 21·6	0·7119	*n*
	9 34 51	,,	+0 48·3	0·0	− 4 16·9	−0·2	2	...	9·7461	...	0·7119	*o*
	9 34 51	,,	−0 27·0	0·0	− 0 14·5	0·0	2	...	9·7461	...	0·7119	*p*
	9 34 51	,,	−0 44·5	0·0	− 0 43·9	0·0	2	...	9·7461	...	0·7119	*q*
	9 48 14	,,	−0 12·5	0·0	+ 6 11·7	+0·2	2	...	9·7492	...	0·6924	*r*
27	9 37 25	A. D.	+0 0·3	0·0	+ 2 23·2	+0·1	3	...	9·7969	...	0·5908	*s*
	9 46 17	,,	−0 31·2	0·0	− 2 13·5	−0·1	3	...	9·7960	...	0·5716	*t*
	9 55 41	,,	−1 14·2	0·0	− 4 19·9	−0·1	3	19 50 28·2	9·7944	35 55 38·0	0·5499	*u*
29	10 13 34	H.	−0 31·9	0·0	− 2 45·0	0·0	2	19 38 55·1	9·7996	34 4 35·9	0·4164	*v*
	10 16 33	,,	+0 41·3	0·0	+ 5 21·1	+0·1	1	19 38 54·0	9·7980	34 4 24·9	0·4068	*w*
30	10 13 34	T.	−1 26·8	0·0	− 3 6·5	−0·1	3	...	9·8044	...	0·3647	*x*
	10 14 45	,,	−1 1·3	0·0	− 4 30·8	−0·1	8	19 32 44·9	9·8032	33 11 3·1	0·3586	*y*
	10 16 45	,,	−0 56·5	0·0	+ 1 58·9	0·0	2	...	9·8023	...	0·3529	*z*
31	10 1 56	A. D.	+2 25·5	0·0	+ 7 53·4	+0·2	3	19 26 15·0	9·8154	32 19 45·2	0·3468	*α*
	10 1 56	,,	−0 6·5	0·0	+ 8 46·3	+0·2	3	...	9·8154	...	0·3468	*β*
	10 14 19	,,	−1 31·0	0·0	+ 3 11·9	+0·1	3	...	9·8072	...	0·2998	*γ*
June 6	10 11 0	H.	+3 25·5	0·0	− 8 42·6	−0·2	4	18 38 43·0	9·7873	27 48 8·8	9·3230	*δ*
	10 12 27	,,	+2 53·3	0·0	− 5 38·4	−0·1	4	18 38 42·9	9·7840	27 48 5·2	9·3029	*ε*

Assumed Mean Places of Comparison Stars.

	Star's Name.	R.A. 1890·0. h m s	N.P.D. 1890·0. ° ′ ″	Authority.
a	B.D. + 42°, No. 3820	20 36 17	47 46	Bonn Observations, vol. v.
b	B.D. + 42°, No. 3821	20 36 20	47 37	„ „ „
c	W.B. (2) XX. 1132	20 34 40·41	47 32 54·2	Weisse's Bessel (2).
d	B.D. + 42°, No. 3815	20 35 44	47 42	Bonn Observations, vol. v.
e	B.D. + 44°, No. 3506	20 30 8	45 35	„ „ „
f	B.D. + 44°, No. 3510	20 30 52	45 33	„ „ „
g	B.D. + 44°, No. 3509	20 30 49	45 45	„ „ „
h	Radcliffe, 4811	20 25 10·47	44 38 49·7	Radcliffe
i	Radcliffe, 4816	20 25 30·25	44 37 55·5	„
j	B.D. + 47°, No. 3091	20 19 31·19	42 47 39·8	Bonn Observations, vol. vi.
k	B.D. + 47°, No. 3095	20 20 39	42 36	„ „ vol. v.
l	Oeltz. Arg. (N) 20427–8	20 18 21·25	41 43 2·1	Oeltz. Arg. (N).
m	Oeltz. Arg. (N) 20397	20 16 43·82	41 43 29·9	„ „
n	Oeltz. Arg. (N) 20296–7	20 12 32·57	40 54 15·3	„ „
o	B.D. + 49°, No. 3237	20 12 36	40 49	Bonn Observations, vol. v.
p	Anonymous			
q	B.D. + 49°, No. 3246	20 14 6	40 46	Bonn Observations, vol. v.

	Star's Name.	R.A. 1890·0. h m s	N.P.D. 1890·0. ° ′ ″	Authority.
r	B.D. + 49°, No. 3243	20 13 36	40 39	Bonn Observations, vol. v.
s	B.D. + 54°, No. 2241	19 50 20	35 53	,, ,, ,,
t	B.D. + 53°, No. 2313	19 50 51	35 57	,, ,, ,,
u	Oeltz. Arg. (N) 19786	19 51 40·90	35 59 43·6	Oeltz. Arg. (N).
v	Oeltz. Arg. (N) 19566	19 39 25·21	34 7 7·4	,, ,,
w	Oeltz. Arg. (N) 19542	19 38 12·70	33 58 50·2	,, ,,
x	Anonymous			
y	Oeltz. Arg. (N) 19459	19 33 44·29	33 15 21·0	Oeltz. Arg. (N).
z	B.D. + 56°, No. 2270	19 33 43	33 10	Bonn Observations, vol. v.
α	Groombridge, 2852	19 23 47·46	32 11 39·3	Radcliffe.
β	B.D. + 57°, No. 2008	19 26 19	32 10	Bonn Observations, vol. v.
γ	B.D. + 57°, No. 2013	19 27 40	32 16	,, ,, ,,
δ	Oeltz. Arg. (N) 18486	18 35 14·90	27 56 43 8	Oeltz. Arg. (N).
ε	Oeltz. Arg. (N) 18495	18 35 46·90	27 53 35·8	,, ,,

Notes.

May 15. Comet very bright.

May 18. Comet very faint.

May 30. Comet rather faint.

The observations are corrected for refraction, but not for parallax.
The initials A.D., T., L., and H. are those of Mr. Downing, Mr. Thackeray, Mr. Lewis, and Mr. Hollis respectively.